TEST EXAM FTV1 – RUG (1)
May 2018

Maximum # points for each question are indicated

Question 1

Consider stirring in a closed reactor vessel filled with water. At a rotational speed of 400 s^{-1} the flow regime changes from laminar to turbulent. This so-called critical rotational speed (n_{cr}) for inducing turbulent flow is a function of the density (ρ) and viscosity (η) of the fluid and the diameter (D) of the vessel.
Density of water: $\rho=10^3$ kg.m^{-3}; Viscosity of water: $\eta=10^{-3}$ kg.m^{-1}.s^{-1}

A) Derive the dimensionless group containing n_{cr}, ρ, η and D. **10**

We repeat the experiment with a geometrically similar but twice as large vessel and with the vessel filled with a different fluid. It turns out that n_{cr} remains the same.

B) What is the numerical value of ρ/η of the fluid? **10**

Question 2

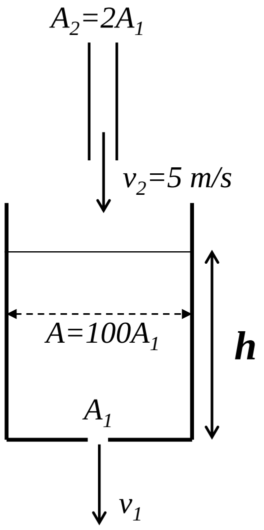
Consider a completely filled cylindrical chemical reactor of length 3 m and diameter 1 m. Due to the chemicals present, exothermal reactions produce a heat per unit volume (q) of 2 kW/m^3. The reactor is not stirred implying that heat transfer inside the reactor occurs by means of conduction only. The reactor is cooled at the outside by flowing air of 20 ^{0}C and the overall heat transfer coefficient between reactor and air equals 100 W/m^{2}K. Assume steady-state conditions.

A) Give the expression for the temperature distribution in the reactor. **10**

B) Given a thermal conductivity of the medium of λ=0.2 W/mK, calculate the temperature at the reactor wall (T_w). **10**

Question 3

Imagine a cylindrical open tank with cross-sectional area A filled with pure water. Because of a hole in the bottom with cross-sectional area A_1 water is flowing out with velocity v_1. Simultaneously, pure water is flowing in with velocity v_2 through a pipe with cross-sectional area A_2 above the tank. Friction losses can be ignored and $A=100A_1$ and $A_2=2A_1$.
Gravitational acceleration (g) is 10 m/s^2.



A) Calculate the steady-state water level height h in the tank, given $v_2=5$ m/s. **10**

While at steady-state ($dh/dt=0$), at $t=0$ compound c is added to the inflow stream with concentration c_{in}. The tank is well-stirred.

B) Give the concentration of c in the tank as function of time given $c_{in}=10$ mol/l. Check your answer by studying the initial and final concentration of c in the tank! **10**

Question 4

A small spherically-shaped solid object (diameter d=1 mm) is dropped into a large volume of water of where it almost immediately reaches a low and constant velocity. Because of the low velocity the flow around the sphere is laminar.
Density of water: $\rho=10^3$ kg.m^{-3}; Viscosity of water: $\eta=10^{-3}$ kg.m^{-1}.s^{-1}, g=10 m.s^{-2}

A) Write down the force balance for the system. **10**

B) Calculate the velocity of the object given a density difference between object and water of 20 kg.m^{-3}. **10**

<u>Question 5</u>

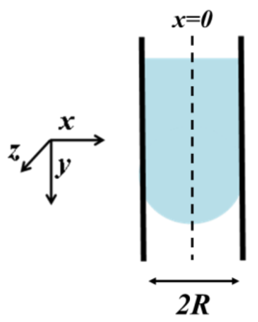
Consider a vertical cylindrical tube with radius *R*. The tube is open at both ends and filled with glycerin, a highly viscous fluid.

Due to gravity, the glycerin flows downwards but because of its high viscosity, at a low and uniform velocity and only in the *y*-direction, implying $v_x = v_z = 0$.

Gravitation is only in the *y*-direction, implying $g_x = g_z = 0$. There are no pressure gradients in any direction. Assume steady-state conditions.

A) What is the shear stress in the center of the pipe at $x=0$? **10**

B) Derive the velocity profile for laminar flow using the Navier-Stokes equations below, i.e. eliminate all irrelevant terms and solve the remaining differential equation. **10**



$$\rho\left\{\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right\}=\eta\left\{\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right\}-\frac{\partial p}{\partial x}+\rho g_x$$

$$\rho\left\{\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right\}=\eta\left\{\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right\}-\frac{\partial p}{\partial y}+\rho g_y$$

$$\rho\left\{\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right\}=\eta\left\{\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right\}-\frac{\partial p}{\partial z}+\rho g_z$$

Final mark = total # points / 10