# Final Examination of Physical Systems

**April 10, 2017, 9:00-12:00**

**INSTRUCTIONS (READ THIS CAREFULLY)**

1. **Make sure you solve each problem on a separate sheet. Solutions of different problems appearing on the same sheet may be discarded.**
2. Write your name and student number clearly on top of every sheet.
3. Solutions which are not on the right sheet will not be graded. If you need extra paper, use the sheets provided and indicate for which problem the extra sheet is used for.
4. Solve the problems in a systematic way and check your answers. If you think you have made a mistake in a calculation, indicate this. Argue then how you intended to get the right answer and write this down.
5. This is a closed-book examination. No books, notes or graphical calculators may be used during the examination.
6. **Write in a legible manner. Unreadable text will not be handled during grading.**

.

**Problem #1. Complex Numbers and Differential Equations (10 Points)**

1. Let us consider the following two complex numbers:

$$X = \exp(2 + i)$$

$$Y = \exp(5i)$$

   0) **USE A NEW SHEET!**
   a) What are the norms of $X$ and $Y$? **(1 point)**
   b) Rewrite $X$ and $Y$ in the form of $a + bi$. **(1 point)**
   c) Calculate $V = X \cdot Y$ and W=X/Y. Write the final answer in the form of $a + bi$. **(2 points)**
   d) What is the complex conjugate of $V$ (=$V^*$)? **(1 point)**
   e) Draw $V$, $V^*$, $W$, $X$ and $Y$ in the complex plane. **(1 point)**

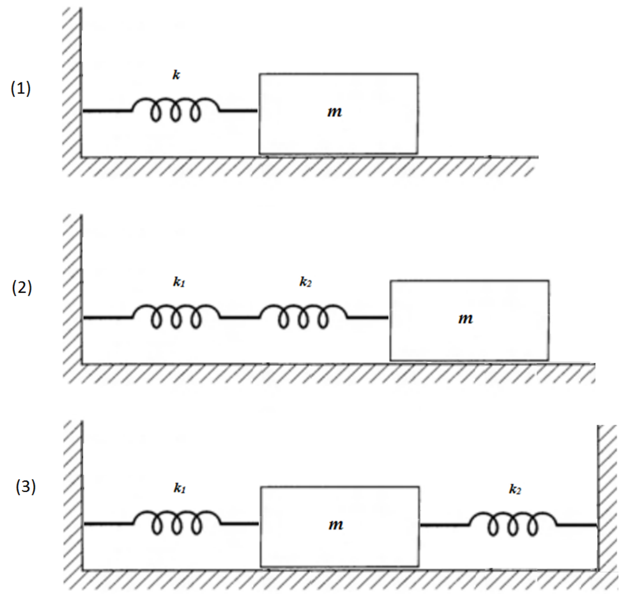
2. Consider the following system:

$$mx''(t) + cx'(t) + kx(t) = 0$$

where $x$ is the displacement of the mass, $m$, and $k$ and $c$ are constants. Calculate the general solution for this. Take $m = 1$, $c = 8$ and $k = 15$. For the boundary conditions, use $x(0) = 0$ and $x'(0) = 8$. **(4 points)**

**Problem #2. Mechanical Oscillations without Damping (17 Points)**

A mass of 500 g is connected in three different configurations (see the figure). For every configuration, assume there is no friction and the mass is put into a harmonic oscillation with an amplitude of 50 cm and a maximum acceleration of 5 m/s$^2$. The motion is described by $x(t) = A\sin(\omega t)$.




0) **USE A NEW SHEET!**

a) Assuming that the effective spring constant is the same (say $k$) for all three configurations above, give the general force balance and the differential equation that applies to all three configurations. **(4 points)**

b) Show that $x(t) = A\sin(\omega t)$ is the solution to the differential equation. **(3 points)**

c) Calculate the spring constant $k$ in the first (1) configuration of the figure shown above. **(2 points)**

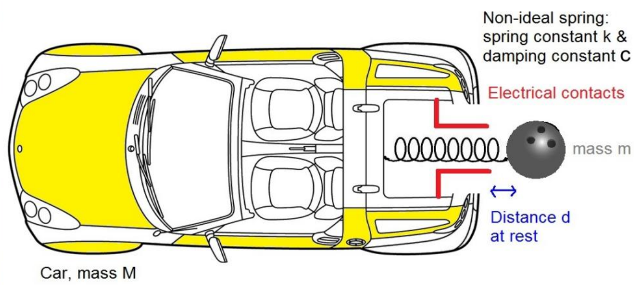
d) Assume that $k_1 = 0.5k_2$. Calculate the spring constants $k_1$ and $k_2$ for the second (2) configuration of the figure shown above. **(4 points)**

e) Again assume $k_1 = 0.5k_2$. Calculate the spring constants $k_1$ and $k_2$ for the third (3) configuration of the figure shown above. **(2 points)**

f) Calculate the maximum speed of mass $m$ in any of the configurations. **(2 points)**

## Problem #3. Mechanical Oscillations with Damping (17 Points)

Mr. Bean does not trust the airbag systems of ordinary English cars. Therefore, he decides to design his own safety system to activate the airbag. He mounts a spring horizontally in the trunk of his car with a metal bowling ball attached to it; see the figure. If the car then suddenly comes to rest, the bowling ball will continue to move forward, compressing the spring. If the spring is compressed far enough, the metal bowling ball will make contact between two electrical poles and the airbag fires. The bowling ball also experiences friction when sliding over the bottom of the trunk.


| | |
|---:|:---|
| Car mass | $M = 1000$ kg |
| Bowling ball mass | $m = 10$ kg |
| Spring constant | $k = 1000$ N/m |
| friction constant | $c = 100$ N $\cdot$ s/m |
| Maximum braking force of the car | $F_{brake} = 5000$ N |

Table 1: Numerical data on the safety system of Mr. Bean

**Hint:** *Apart from the spring force and friction, the bowling ball experiences another force. This force is called an "apparent force" and works in the opposite direction of the spring force. The origin of this force is the inertia of the bowling ball: if the car starts to decelerate, the bowling ball wants to keep its original velocity. This behaviour acts in the same way as a force. Hence, the name "apparent force". If the total force on the car (due to the engine, brakes, etc.) equals $F(t)$, the magnitude of the apparent force on the bowling ball equals $(m/M) \cdot F(t)$. This force should then be treated as one of the forces acting on the ball.*

0) **USE A NEW SHEET!**

a) Given that the airbag should not be activated by the breaking force of the car, calculate the minimal required distance between the electrical poles and the bowling ball (in the situation that the spring is uncompressed). Balance the spring force with the apparent force to obtain this answer. **(3 points)**

b) Draw a picture of the safety system and specify all relevant forces on the Bowling ball in this picture. **(2 points)** Derive the differential equation of the motion of the bowling ball. **(2 points)**

*Mr. Bean is driving at a constant speed of* 50 *km/h when he hits another car in a frontal collision. Therefore, Mr. Bean immediately comes to a full stop at the moment of the collision. In this case, the bowling ball will start its motion from the equilibrium position with the speed that the car had just before the collision. Since at all times (except at the moment of the collision) the force on the car is zero, the motion of the car will be an undriven damped harmonic oscillation. This motion is the transient solution of the differential equation you obtained in part b) and is described by:*

$$x(t) = Be^{-\gamma t}\cos(\omega t - \alpha)$$

c) Calculate $\gamma$ and $\omega$ (different from $\omega_0$). Use the fact that $x(t)$ is a solution of the differential equation obtained in part b) and $2\gamma = \frac{c}{m}$. **(2 points)**

d) Calculate $B$ and $\alpha$. Clearly state which initial conditions you use for this calculation. **(2 points)**

e) Will Mr. Bean survive the collision? Assume that he only survives if the airbag fires. This will only happen if the first maximum for $t > 0$ is bigger than the distance obtained in part a) (in absolute sense). Calculate whether this happens or not. **(2 points)**

*Mr. Bean is driving through a long street filled with speed bumps. As a result, the force on the car is described by $F(t) = F_0 \cos\omega t$ with $F_0 = 4500$ N. Therefore, the motion of the bowling ball will now be a driven damped harmonic oscillation (after the initial effects have dampened out). In this situation, the motion of the bowling ball is described by:*

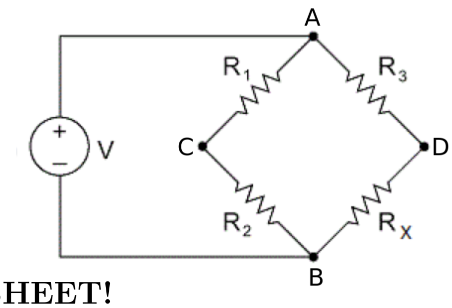
$$x(t) = \frac{F_0/M}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \cdot \cos(\omega t - \phi) \qquad \tan\phi = \frac{2\gamma\omega}{\omega^2 - \omega_0^2}$$

f) Calculate for which range of frequencies $\omega$ the airbag will activate. Again, assume that the airbag only activates if the amplitude of $x(t)$ is bigger than the distance you calculated in part a). **(2 points)**

g) Given that the distance between 2 succeeding speed bumps is 50 m, is it smart for Mr. Bean to drive the maximum speed of 30 km/h? Find out whether this maximum speed introduces a frequency $\omega$ within the range calculated in the previous part. Assume the time it takes to travel between the two bumps to be equal to the time for one oscillation. **(2 points)**

**Problem #4. Electric Circuit – I (18 Points)**

The circuit given below is called a Wheatstone bridge, named after Charles Wheatstone who helped popularize the configuration. It is used as a test circuit to determine the resistance of $R_x$. In the example below the symbol $V$ is used to denote the voltage of the source.




0) **USE A NEW SHEET!**

a) Give an expression for the total resistance of the circuit. **(2 points)**

b) Give the Kirchhoff's Current Law equations for nodes A and B. **(2 points)**

c) Use Kirchhoff's Voltage Law to determine the voltages at C and D. To make this easy, assume that the voltage at B is 0 and $V_1$ and $V_2$ are the potential drops between points C and B, and D and B, respectively. **(4 points)**

d) In order to measure the resistor $R_x$, it is best to use a voltmeter to measure the voltage across $V_1$ and $V_2$, assume that $V_1$ is connected to the positive terminal on the meter. The voltage between the terminals of the voltmeter is clearly $V_m = V_1 - V_2$. Now express $V_m$ in terms of $V$. **(2 points)**

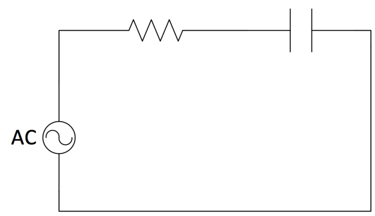
e) The voltmeter reads 1 Volt; given $R_1 = 1$ k$\Omega$, $R_2 = 5$ k$\Omega$, $R_3 = 10$ k$\Omega$, and $V = 10$ V, calculate the value of $R_x$. **(3 points)**

f) What are the largest and smallest potential differences that can be measured between $V_1$ and $V_2$? *HINT: use the smallest and largest possible values for $R_x$.* **(3 points)**

*By adjusting $R_2$, one can then easily obtain the value of the unknown $R_x$ with the condition that these two voltages are the same and, subsequently, no current flows over from C to D even if a bridge is placed between them. This is the principle of Wheatston bridge.*

g) If the difference between $V_1$ and $V_2$ is 0, there is a simple relation relating the four resistances. Using the result of part d) or simple reasoning, obtain this relationship. **(2 points)**

**Problem #5. Electric Circuit – II (18 Points)**




Consider the above circuit.

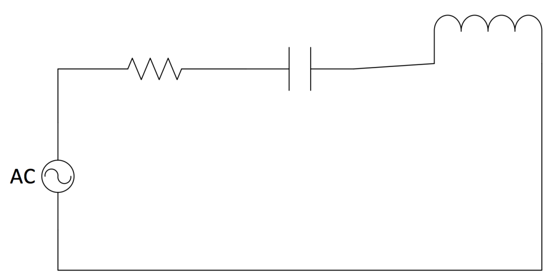
0) **USE A NEW SHEET!**

a) Give the expression for the complex impedance of the capacitor, $C$? **(2 points)**

b) Calculate the complex transfer function, namely the ratio between the voltage over the capacitor and the input voltage, $A_C$ for $C$ (HINT: use the generalized Ohm's law)? **(4 points)**

c) Calculate the norm and the phase of this transfer function. **(2 points)**

d) Is this a high pass or a low pass filter? Show this mathematically by considering $\omega \ll (RC)^{-1}$ and $\omega \gg (RC)^{-1}$ . **(2 points)**




Consider the circuit above with $R = 1.2\ \Omega$, $L = 180$ mH, $\phi_{A_C} = \pi/4$, with a frequency of the power supply of 50 Hertz.

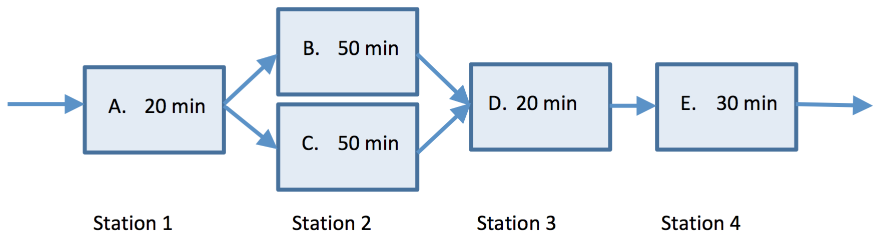
e) Give the expression for the complex impedance of the inductor, $L$? **(2 points)**

f) Using the specifications of the circuit above, calculate the value of $C$. (HINT: derive the phase of $|A_C|$ first and note that this is different from the one obtained in part b))? **(4 points)**

g) Calculate the resonance frequency (ignore the damping in the system)? **(2 points)**

**Problem #6. Business Dynamics (20 Points)**

Consider the following production line depicted below. The line runs in a single piece flow. The processing times per product per machine are also indicated in the figure. Station 2 consist of identical machines B & C.




0) **USE A NEW SHEET!**

a) Which station is the bottleneck and what is the corresponding bottleneck rate, $r_b$? **(2 points)**

b) What is the critical Work-In-Progress (WIP)? **(3 points)**

c) Machine C in station 2 breaks down. Does this influence the throughput? Explain by calculating the throughput before and after breakdown and showing that they are similar/different. **(3 points)**

d) If we take into consideration that the system is empty when starting, how many complete products get produced in a working day of 8 hours, with machine C broken the whole day? **(4 points)**

The bullwhip effect is a trend of larger and larger swings in inventory as one looks at companies further back in the supply chain of a product.

e) Name 3 possible causes for the bullwhip effect and explain how this could lead to the bullwhip effect. **(3 points)**

Oscillations in a supply chain can be described by the following formula:

$$\frac{d^2q_i(t)}{dt^2} + \frac{\beta+\epsilon}{T}\frac{dq_i(t)}{dt} + \frac{1}{T\tau}q_i(t) = \frac{1}{T}\left(\frac{1}{\tau}q_{i+1}(t) + \beta\frac{dq_{i+1}(t)}{dt}\right) \qquad (1)$$

f) Which part of this equation represents the driving factor (force) and what does it present in the business dynamics? **(2 points)**

g) Which part of this equation represents the damping factor? **(1 point)**

h) What is the condition for having the bullwhip effect (oscillations)?**(2 points)**