Re-Examination of Physical Systems

June 30, 2017, 9:00-12:00

INSTRUCTIONS (READ THIS CAREFULLY)

1. **Make sure you solve each problem on a separate sheet. Solutions of different problems appearing on the same sheet may be discarded.**
2. Write your name and student number clearly on top of every sheet.
3. Solutions which are not on the right sheet will not be graded. If you need extra paper, use the sheets provided and indicate for which problem the extra sheet is used for.
4. Solve the problems in a systematic way and check your answers. If you think you have made a mistake in a calculation, indicate this. Argue then how you intended to get the right answer and write this down.
5. This is a closed-book examination. No books, notes or graphical calculators may be used during the examination.
6. **Write in a legible manner. Unreadable text will not be handled during grading.**

Problem #1. Complex Numbers and Differential Equations (10 Points)

Part 1) Consider the following complex numbers:

a. $R = (2+i)(3+i)$

b. $S = (2+i)^2$

c. $T = \exp(2\pi i)$

Perform the following operations on these numbers:

0) **USE A SEPARATE SHEET!**

i) Write R, S and T in $a + bi$ form. **(1.5 points)**

ii) Determine the modulus and phase for every one of them. **(3 points)**

iii) Draw R, S and T in the complex plane. **(1.5 point)**

Part 2) Consider the following differential equation:

$$y''(t) + 25y(t) = 0$$

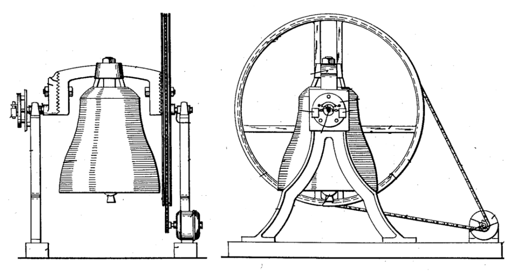
with initial conditions $y(0) = 2$ and $y'(0) = 5$. Give the general solution in the form of cosine and sine with the right coefficients and parameters. **(4 points)**

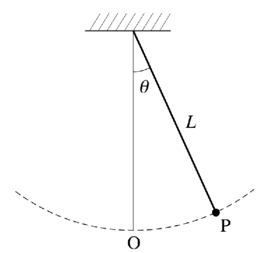
Problem #2. Mechanical Oscillations without Damping (17 Points)

We will try to approximate the swinging motion of a church bell by describing it as a mathematical pendulum depicted in the figure below. The mathematical pendulum consists of a massless rigid bar, with a length, L, of 2.5 m. The mass m of the bell is concentrated in point P with a total of 7850 kg. The equation of motion is given by:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin(\theta) = 0$$

where $g = 9.81$ m/s^2 and the amplitude is given by $\theta = 20°$.





0) **USE A SEPARATE SHEET!**

a) Give the assumption under which the equation of motion can be simplified into the following equation. **(1 point)** Express ω_0 in terms of the known parameters. **(1 point)**

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0$$

b) Show that $\theta(t) = \theta_0 \sin(\omega_0 t + \alpha)$ is the solution to this differential equation. **(3 points)**

c) Calculate the natural frequency and the period of the oscillation. **(2 points)**

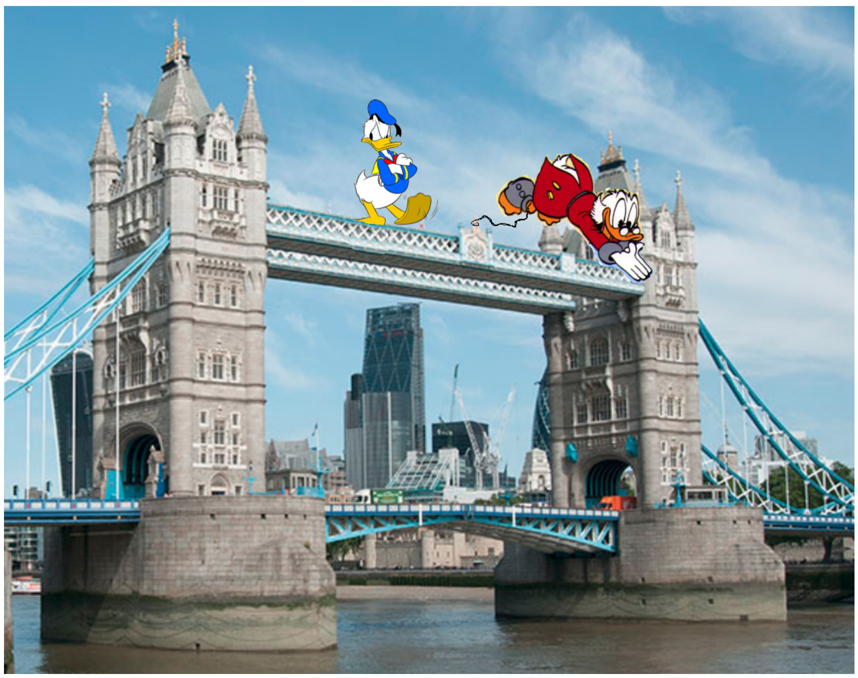
d) Calculate the maximum potential energy of the bell. **(4 points)**

e) Give the expression for the total energy of the system at any point. What is the value of the total energy? Also, give the position at which the bell has its maximum speed and the position at which it has its maximum acceleration. **(4 points)**

f) What would happen to the period, the maximum speed, and the maximum acceleration of the bell if we decrease the total weight to half its value (3925 kg)? **(2 points)**

Problem #3. Mechanical Oscillations with Damping (17 Points)

Donald Duck has taken a bet with his uncle Dagobert whether he can make a successful bungee jump from the London Tower Bridge. If Dagobert cannot make the jump without hitting the water, Donald does not have to pay rent for his house. The figure below shows how Dagobert is going to make the jump. Numerical data on the jump is given in table below the figure.



Dagoberts body mass	$m = 40$ kg
The rope can be considered massless	
Spring constant of the rope	$k = 8$ N/m
Jump height above the water	$h = 200$ m
Rope length in its unstrechted form	$l = 50$ m

0) **USE A SEPARATE SHEET!**

a) Assume Dagobert makes a free fall from the bridge. Calculate how close Dagobert will get to the water by balancing Gravitational potential energy and spring potential energy. Will Dagobert hit the water? *HINT: Make a drawing and label all the distances to get a clearer picture.* **(3 points)**

After Dagoberts initial free fall, he will start to oscillate between the water and the bridge in much the same way as a mass-spring system.

b) Calculate how high Dagobert will be above the water after the oscillation has damped out completely (This is Dagoberts equilibrium position). *HINT: Use force balance in equilibrium.* **(1 point)**

If the answer of part a) is unknown to you, assume from here on that Dagobert swings up to 10 *m above the water. If the answer of part b) is unknown to you, assume from here on an equilibrium height of* 100 *m.*

During Dagoberts first swing back to the bridge, the highest point he reaches is 20 m below the bridge.

c) Calculate the damping factor γ. For this calculation, you may assume that γ is sufficiently small so that Dagobert oscillates with his natural frequency. **(3 points)**

d) Calculate the frequency of Dagoberts damped oscillation. Justify the assumption of the previous question that Dagobert oscillates with (almost) his natural frequency. **(1 point)**

e) Calculate the maximum speed Dagobert will ever have during his entire jump. **(5 points)**

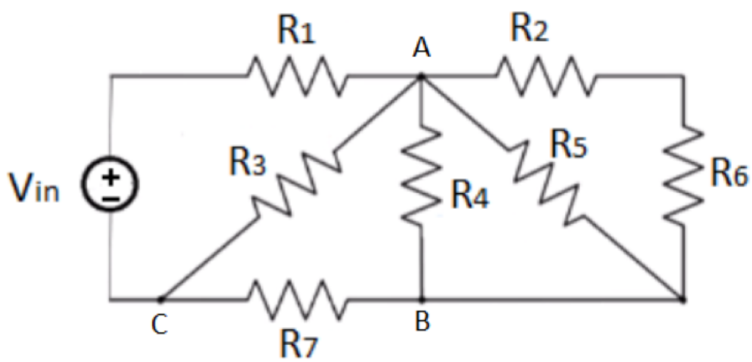
Donald Duck tries to sabotage the jump by pulling the rope at the bridge. His pulling force equals $F_0 \cos(\omega t)$ with $F_0 = 100$ N. This causes Dagobert to undergo a driven damped harmonic oscillation. If $x(t)$ is the deviation from the equilibrium position of part b), the steady state solution of this oscillation equals:

$$x(t) = A\cos(\omega t - \theta) \qquad A = \frac{F_0/m}{\sqrt{(\omega^2 - {\omega_0}^2)^2 + 4\gamma^2\omega^2}} \qquad \tan(\theta) = \frac{2\omega\gamma}{\omega^2 - {\omega_0}^2}$$

f) Calculate the lowest driving frequency ω Donald has to use to make sure that Dagobert hits the water. **(4 points)**

Problem #4. Electric Circuit – I (18 Points)

All calculations and equations for this problem are based on the circuit drawn below. The input voltage is 10 Volts.



0) **USE A SEPARATE SHEET!**

a) Write down the general form of Kirchhoff's laws for current and voltages for any circuit. **(2 points)**

b) Give the three Kirchhoffs Current Law equations for the nodes A, B, and C. **(3 points)**

c) Give four Kirchhoffs Voltage law equations corresponding to the above circuit. *Hint: Pick any loop that contains any of the three points or a combination of them and draw a loop with a direction. An example containing the three points is the triangle ABC with the loop which goes clockwise around it.* **(4 points)**

d) Calculate the equivalent resistance of the circuit if $R_1 = 5\,\Omega$, $R_2 = 3\,\Omega$, $R_3 = 10\,\Omega$, $R_4 = 20\,\Omega$, $R_5 = 10\,\Omega$, $R_6 = 7\,\Omega$, and $R_7 = 6\,\Omega$.
Hint: To avoid confusion, simplify the circuit step by step. **(3 points)**

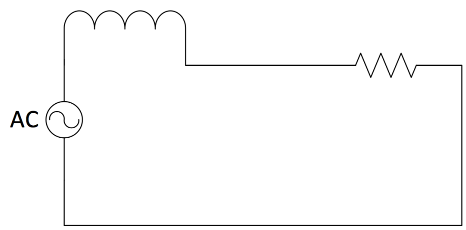
e) Calculate the current through one of the resistors. **(2 points)**

f) Calculate the voltage over one of the resistors. **(2 points)**

g) Now, assume the input voltage source is non-ideal and has an internal resistance of 20 Ω which can be considered as an extra resistance in your circuit. Recalculate the current and the voltage that you calculated in e) and f).
(2 points)

Problem #5. Electric Circuit – II (18 Points)

Consider the following AC-circuit:



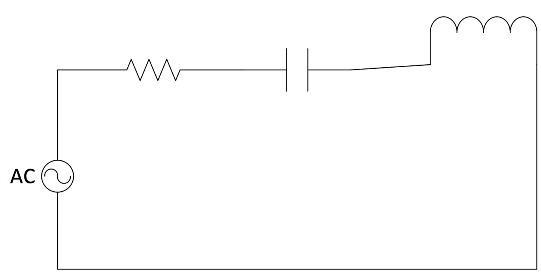
0) **USE A SEPARATE SHEET!**

a) What is the total complex impedance of this system? **(2 points)**

b) Give an expression (possibly containing complex numbers) for the transfer function $A_L = \frac{V_L}{V_{in}}$, over L? If you have a complex number, make sure you express it in one of the standard forms. **(4 points)**

c) Calculate the modulus and the phase of A_L. **(4 points)**

Now, consider the following circuit:



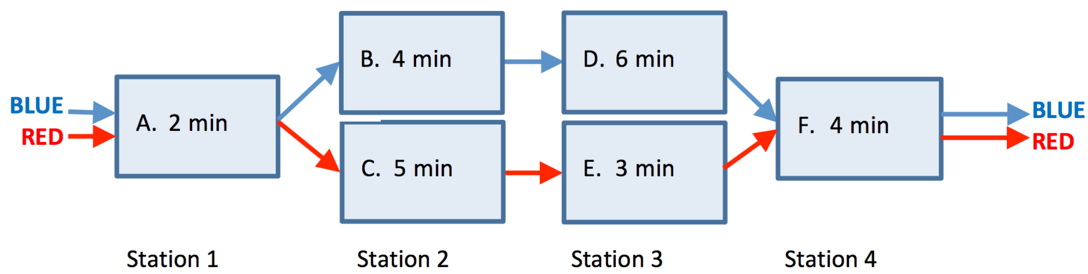
In this circuit, $R = 120\ \Omega$, $L = 0.8$ mH.

d) Calculate the analytical form of $|A_R|$ for this circuit. **(6 points)**

e) The AC power source in the figure comes from the net voltage that you also have at home. Under the assumption that $|A_R| = \frac{1}{2}$, calculate the value of the capacitor, C. **(2 points)**

Problem #6. Business Dynamics (20 Points)

In a certain company they produce two different products: product BLUE (above) and product RED (below). Consider the following production lines depicted below. Some of the process steps require the same machine for both products.



We consider that machines A and F can produce the products RED and BLUE simultaneously. So, every 2 minutes machine A produces both a RED as well as a BLUE product (similar to machine F).

0) **USE A SEPARATE SHEET!**

a) Which machine is the bottleneck for the RED product? And which machine for BLUE? **(2 points)**

b) How many products RED and how many BLUE get produced per hour? Clearly indicate which production rate corresponds to which product. **(3 points)**

c) Name two ways on how you can increase the throughput for the BLUE product and explain why this increases the throughput. **(2 points)**

d) What is the utilization rate of machine E? **(2 points)**

We now consider that machines A and F cannot produce RED and BLUE simultaneously. In this case, the products will enter the production line in alternating order. (RED-BLUE-RED-BLUE etc.). Machine F will produce any product ready, but in case both RED and BLUE are ready, it will produce these in alternating order.

e) What is in this case the throughput for the RED product? And what is it for the BLUE product? **(3 points)**

Just like in the mechanical and electrical domain, oscillations can occur in the business domain as well.

f) Name a well-known type of oscillation in Business Dynamics and describe its behavior. *Hint: Make use of a graph to clearly describe its behavior.* **(3 points)**

g) Name one cause of this well-known type of oscillation and name one measure to prevent it. Also clearly explain why this causes and prevents this type of oscillation. **(2 points)**

h) Oscillations in a supply chain can be described by the following formula:

$$\frac{d^2 q_i(t)}{dt^2} + \frac{\beta + \epsilon}{T}\frac{dq_i(t)}{dt} + \frac{1}{T\tau} q_i(t) = \frac{1}{T}\left(\frac{1}{\tau} q_{i+1}(t) + \beta \frac{dq_{i+1}(t)}{dt}\right) \quad (1)$$

Which part of the equation represents the eigen frequency? **(1 point)**

i) Which condition should be imposed in the system to realize an over-damped situation? **(2 points)**