1 General equalities

Spherical symmetric part of tensor: (Captures *change in volume* for ε)

$$\mathbf{A}^s = \frac{1}{3}tr(\mathbf{A}) * \mathbf{I} \tag{1}$$

Deviatoric part of tensor: (Captures *change in shape* for ε)

$$\mathbf{A}^d = \mathbf{A} - \mathbf{A}^s = \mathbf{A} - \frac{1}{3}tr(\mathbf{A}) * \mathbf{I} \tag{2}$$

Gauss Theorem:

$$\int_V \nabla \mathbf{v} dV = \int_S \mathbf{v} \cdot \mathbf{n} dS \tag{3}$$

Angle between vectors:

$$\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}||\,||\mathbf{w}||cos(\angle \mathbf{vw}) \tag{4}$$

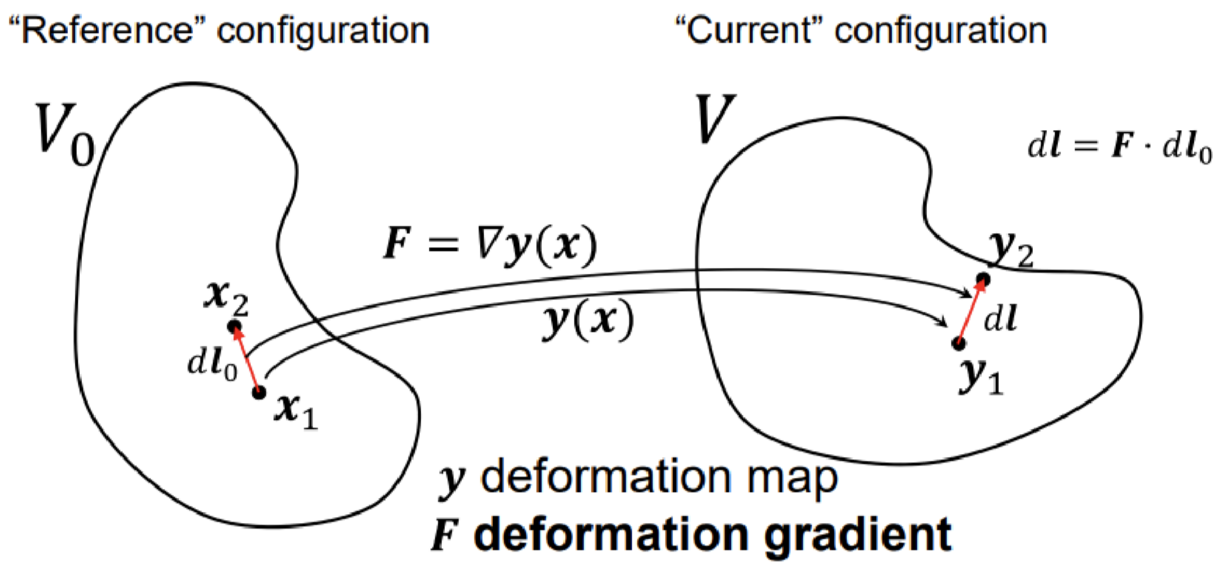
ABC-formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{5}$$

Rotation:

$$\mathbf{R} = \begin{bmatrix} cos(\alpha) & sin(\alpha) & 0 \\ sin(\alpha) & -cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad det(\mathbf{R}) = 1,\ \mathbf{R}^T = \mathbf{R}^{-1} \tag{6}$$

2 Small strain:



$$\mathbf{F} = \nabla \mathbf{y} = \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \tag{7}$$

Displacement of point

$$\mathbf{u}(\mathbf{x}) = \mathbf{y}(\mathbf{x}) - \mathbf{x} \tag{8}$$

Displacement gradient:

$$\nabla \mathbf{u} = \nabla \mathbf{y} - \mathbf{I} = \varepsilon + \omega \tag{9}$$

Strain:

$$\varepsilon = \frac{1}{2} * (\nabla\mathbf{u} + \nabla\mathbf{u}^T) \tag{10}$$

Rotation:

$$\omega = \frac{1}{2} * (\nabla\mathbf{u} - \nabla\mathbf{u}^T) \tag{11}$$

Only when $||u|| = \sqrt{u : u} << 1$!

2.1 Invariants:

if eigenvalues of ε are given by ϵ_i.

$$I_{\mathbf{A}} = tr(\mathbf{A}) \tag{12}$$

$$II_{\mathbf{A}} = \epsilon_1 * \epsilon_2 + \epsilon_2 * \epsilon_3 + \epsilon_1 * \epsilon_3 \tag{13}$$

$$III_{\mathbf{A}} = det(\mathbf{A}) = \epsilon_1 * \epsilon_2 * \epsilon_3 \tag{14}$$

3 Kinematics at large deformation

Nanson's Formula (change in area)

$$\mathbf{n}dA = det(\mathbf{F})\mathbf{F}^{-T} \cdot \mathbf{n}_0 dA_0 \tag{15}$$

Change in volume:

$$det(\mathbf{F}) = \frac{dV}{dV_0} \tag{16}$$

Relative change in volume:

$$tr(\varepsilon) = \frac{dV - dV_0}{dV_0} \tag{17}$$

Right Cauchy-Green deformation:

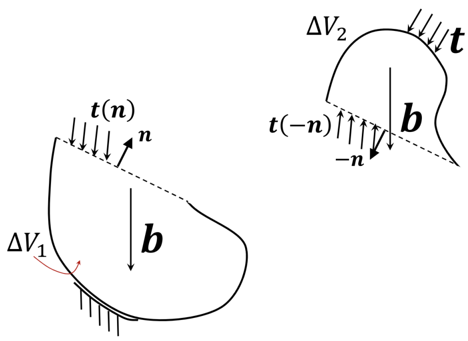
$$\mathbf{C} = \mathbf{F}^T \cdot \mathbf{F} \tag{18}$$

$\sqrt{\mathbf{C}} = \mathbf{U}$ *Pure deformation!*
Polar decomposition ($(Rotation) \cdot (Pure\ deformation)$):

$$\mathbf{F} = \mathbf{R} \cdot \mathbf{U} \tag{19}$$

Find $\mathbf{U}$ using $\sqrt{\mathbf{C}} = PD^{1/2}P^{-1}$, ($P$ is eig. vector matrix of C, D is eig. value matrix of (C). Find $\mathbf{R} = \mathbf{F} \cdot \mathbf{U}^{-1}$
Green Langrange Strain:

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I}) \tag{20}$$



Cauchy's equation:

$$\mathbf{t}(\mathbf{n}) = \sigma \cdot\ \mathbf{n} \quad \Longleftrightarrow \quad t_i = \sigma_{ij} n_j \tag{21}$$

The stress tensor (σ) used can be calculated using:

$$\sigma = \mathbf{t}(\mathbf{n}) \otimes \mathbf{n} \tag{22}$$

The normal stress (σ) is calculated by projecting the vector $\mathbf{t}$ along $\mathbf{n}$ (vector initially chosen)

$$\sigma = \mathbf{t}(\mathbf{n}) \cdot \mathbf{n} \tag{23}$$

The shear stress (τ) is calculated by projecting the vector **t** along a vector along its plane (**s**).

$$\tau = \mathbf{t}(\mathbf{n}) \cdot \mathbf{s} \tag{24}$$

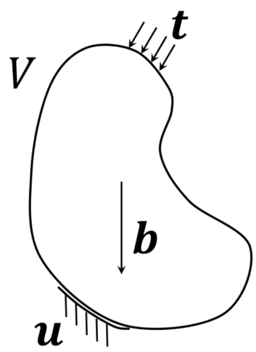
Deviatoric stress:

$$\sigma^d = \sigma - \frac{tr(\sigma)}{3}\mathbf{I} \tag{25}$$

Hydrostatic stress:

$$p\mathbf{I} = \frac{tr(\sigma)}{3}\mathbf{I} \tag{26}$$

4 Stress in small deformations



Balance of forces: (equilibrium of momentum)

$$\int_V \mathbf{b} dV + \int_S \mathbf{t} dS \iff \mathbf{b} + div(\sigma) = 0 \iff b_i + \sigma_{ij,j} = 0 \tag{27}$$

Equilibrium of moment of moments:

$$\sigma = \sigma^T \tag{28}$$

$$\delta W^{ext} = \int_v \mathbf{b} \cdot \delta \mathbf{u} dV + \int_s \mathbf{t} \cdot \delta \mathbf{u} dS \tag{29}$$

$$\delta W^{int} = \int_V \delta\epsilon : \sigma dV \tag{30}$$

$$\delta W^{ext} = \delta W^{int} \tag{31}$$

5 Elasticity in small deformations

$$\sigma = \mathbb{C} : \varepsilon \tag{32}$$

Internal stored energy *density (i.e. per volume)*: (Hooke's law)

$$\psi = \frac{1}{2}\sigma : \epsilon = \frac{1}{2}\varepsilon : \mathbb{C} : \epsilon \tag{33}$$

Stress in a material is the change in internal stored energy over change in strain:

$$\sigma = \frac{\partial \psi}{\partial \varepsilon} \ \left(\iff \ \sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}}\right) \tag{34}$$

Isotropic stiffness tensor (symmetries due to isotropy)

$$\mathbb{C} = \lambda \mathbf{I} \otimes \mathbf{I} + 2\mu \mathbb{I}^s \tag{35}$$

With $\mathbb{I}^s$ being the 4th order symmetric unit tensor.

$$\mathbb{C}_{ijhk} = \lambda \delta_{ij}\delta_{hk} + \mu(\delta_{ih}\delta_{jk} + \delta_{ik}\delta_{jh}) \tag{36}$$

$$[\sigma] = [\mathbb{C}] : [\varepsilon] = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda + 2 \cdot \mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2 \cdot \mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2 \cdot \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} \tag{37}$$

$$\sigma = \lambda tr(\varepsilon)\mathbf{I} + 2\mu\varepsilon \tag{38}$$

4th order compliance tensor ($\mathbb{S}$):

$$\varepsilon = \mathbb{S} : \sigma \tag{39}$$

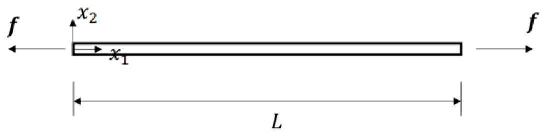
$$\mathbb{S} = -\frac{\lambda}{2\mu(3\lambda + 2\mu)}\mathbf{I} \otimes \mathbf{I} + \frac{1}{2\mu}\mathbb{I}^s \tag{40}$$

$$\varepsilon = -\frac{\lambda}{2\mu(3\lambda + 2\mu)}tr(\sigma)\mathbf{I} + \frac{1}{2\mu}\sigma \tag{41}$$

	(E, ν)	(K, μ)	(E, μ)
E	E	$\frac{9\mu K}{3K + \mu}$	E
ν	ν	$\frac{3K - 2\mu}{2(3K + \mu)}$	$\frac{E - 2\mu}{2\mu}$
G, μ	$\frac{E}{2(1 + \nu)}$	μ	μ
K	$\frac{E}{3(1 - 2\nu)}$	K	$\frac{\mu E}{3(3\mu - E)}$
λ	$\frac{E\nu}{(1 + \nu)(1 - 2\nu)}$	$\frac{3K - 2G}{3}$	$\frac{\mu((E - 2\mu)\,)}{3\mu - E}$

6 Beam Theory

6.1 Uniaxial tension



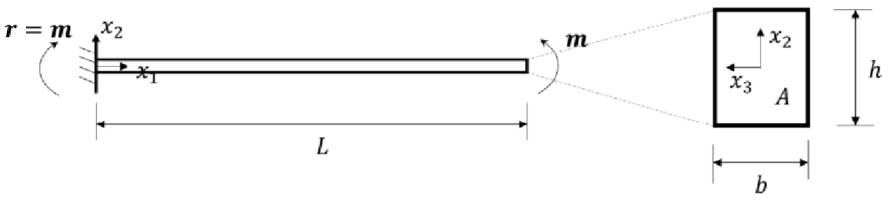
Stress tensor:

$$\sigma = \frac{\mathbf{f}}{A}\mathbf{e}_1 \otimes \mathbf{e}_1 \tag{42}$$

using $\varepsilon = -\nu tr(\sigma)\mathbf{I} + (1+\nu)\sigma$:

$$\varepsilon = \frac{\mathbf{f}}{EA}\mathbf{e}_1 \otimes \mathbf{e}_1 - \frac{\nu}{E}\frac{\mathbf{f}}{A}\mathbf{e}_2 \otimes \mathbf{e}_2 - \frac{\nu}{E}\frac{\mathbf{f}}{A}\mathbf{e}_3 \otimes \mathbf{e}_3 \tag{43}$$

6.2 Uniform Bending



Moment of inertia:

$$I = \frac{bh^3}{12} \tag{44}$$

Curvature is moment over bending stiffness:

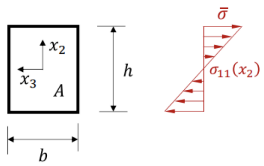
$$\kappa = \frac{1}{R} = \frac{M}{EI} \tag{45}$$

$$\varepsilon_{11} = \kappa x_2 \mathbf{e}_1 \otimes \mathbf{e}_1 \tag{46}$$

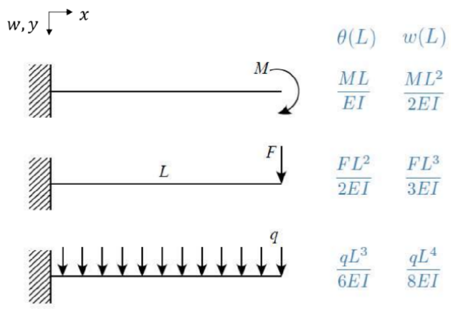
$$\varepsilon = \frac{1}{E}\Big(- \nu tr(\sigma)\mathbf{I} + (1+\nu)\sigma\Big) \tag{47}$$

$$\sigma = E \cdot \kappa x_2 \mathbf{e}_1 \otimes \mathbf{e}_1 \tag{48}$$

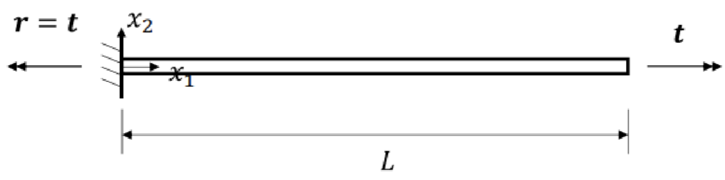
Bend limit:
Since $\sigma_{11} = \kappa E x_2 = \frac{M}{I}x_2, \rightarrow \bar{\sigma} = \frac{Mh}{2I} \leq \sigma_Y$



$$M_{lim} = \frac{\sigma_Y bh^2}{6} \tag{49}$$



6.3 Torsion



Cross sectional shear is total angle over length

$$\varphi = \frac{\phi}{l} \tag{50}$$

Hence rotation at x_1 is $\varphi \cdot x_1$.

$$u_2 = -\varphi \cdot x_1 x_3 \quad u_3 = \varphi \cdot x_1 x_2 \tag{51}$$

Resulting in:

$$[\varepsilon] = \begin{bmatrix} 0 & \frac{-\varphi}{2} x_3 & \frac{\varphi}{2} x_2 \\ \frac{-\varphi}{2} x_3 & 0 & 0 \\ \frac{\varphi}{2} x_2 & 0 & 0 \end{bmatrix} \tag{52}$$

$$[\sigma] = \begin{bmatrix} 0 & \mu \cdot \frac{-\varphi}{2} x_3 & \mu \cdot \frac{\varphi}{2} x_2 \\ \mu \cdot \frac{-\varphi}{2} x_3 & 0 & 0 \\ \mu \cdot \frac{\varphi}{2} x_2 & 0 & 0 \end{bmatrix} \tag{53}$$

Torque:

$$t = \int_A x_2 \sigma_{13} + x_3 \sigma_{12} dA = \varphi \cdot \mu I_p \tag{54}$$

(with $I_p = \frac{1}{2}\pi R^4$)

7 Elastoplasticity

Internally stored energy + applied/dissipated is smaller than zero:

$$\int_v \frac{d\psi}{dt}dV - \int_v P_{ext}(\dot{u})dV = \int_v \frac{d\psi}{dt}dV - \int_v \sigma : \dot{\varepsilon}dV \leq 0 \tag{55}$$

Replace $\frac{d\psi}{dt}$ with derivative w.r.t. its inputs. (ex. $\frac{d\psi}{dt} = \frac{d\psi}{d\varepsilon} \cdot \dot{\varepsilon}$ (might require chain rule)).
($\psi = \psi(\varepsilon_e, \alpha) \rightarrow \frac{d\psi}{dt} = \frac{d\psi}{d\varepsilon_e} \cdot \dot{\varepsilon_e} + \frac{d\psi}{d\alpha} \cdot \dot{\alpha} = \frac{d\psi}{d\varepsilon_e} \cdot \dot{\varepsilon_e} + 0$)

Dissipation (energy leaving per volume unit per time unit) (from definition of internal work) (Can often use $\sigma_{vm} = \sigma_Y$):

$$D = \sigma : \dot{\varepsilon}_p = \sigma_{vm} \cdot \dot{\varepsilon}_{eq} (\geq 0) \tag{56}$$

$$\sigma = \frac{\partial \psi}{\partial \varepsilon_e} \tag{57}$$

Von Mises equivalent stress (*J2*):

$$\sigma_{vm} = \sqrt{\frac{3}{2}\sigma^d : \sigma^d} \tag{58}$$

Von Mises strain:

$$\varepsilon_{eq} = \sqrt{\frac{2}{3}\varepsilon_p : \varepsilon_p} \tag{59}$$

Von Mises criterion: (with σ_i being principal stress (eigenvalue, of σ))

$$f(\sigma) = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 2\sigma_Y^2 = 0 \tag{60}$$

Tresca criterion: (with σ_i being principal stress (eigenvalue, of σ))

$$f(\sigma) = max(|\sigma_1 - \sigma_2|, |\sigma_1 - \sigma_3|, |\sigma_2 - \sigma_3|) - \sigma_Y = 0 \tag{61}$$

1D criterion:

$$f(\sigma) = |\sigma| - \sigma_Y(-\pi) = 0 \tag{62}$$

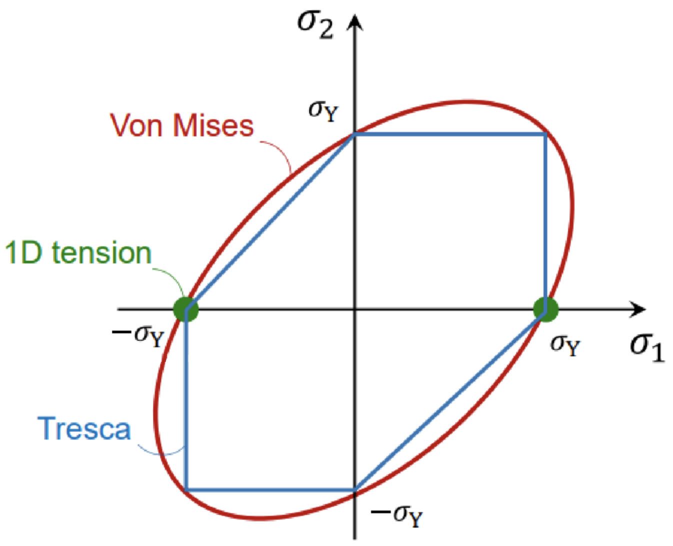


Figure 1: border defines *yield surface*, interior defines *admissibility domain*

Stress (σ) is dependent upon *elastic* strain (ε_e). Plastic deformation (π) is dependent upon *plastic* strain (ε_p) ($\dot{\lambda} = \dot{\gamma} = \dot{\alpha} = |\dot{\varepsilon}_p|$).

$$\dot{\varepsilon}_p = \dot{\gamma}R(\sigma) = \dot{\varepsilon}^{eq}\frac{f(\sigma)}{\sigma} \tag{63}$$

$$\psi(\varepsilon_e, \alpha) = \frac{1}{2}E\varepsilon_e^2 + \frac{1}{2}K\alpha^2 \tag{64}$$

$$\pi = -K \cdot \alpha \tag{65}$$

$$\sigma = \frac{\partial \psi}{\partial \varepsilon_e} = E \cdot \varepsilon_e = E(\varepsilon - \varepsilon_p) \tag{66}$$

7.1 Common hardening laws (*J2*)

Linear hardening:

$$\sigma_Y = \sigma_{Y0} + H\varepsilon^p_{eq} \tag{67}$$

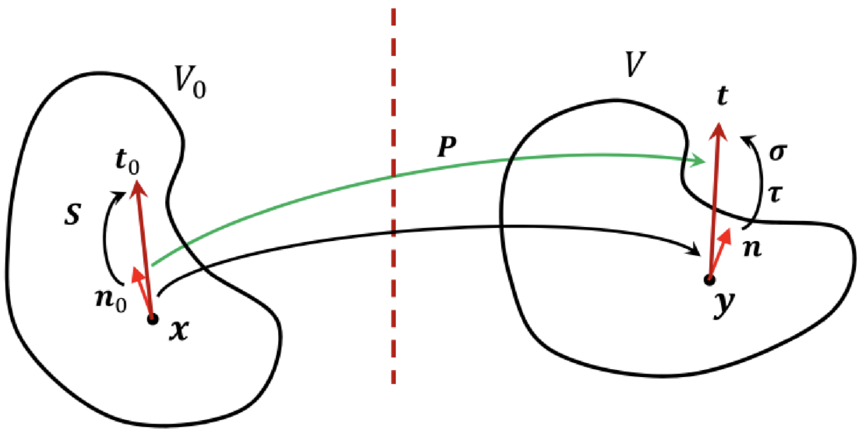
Ludwik law:

$$\sigma_Y = \sigma_{Y0}\Big[1 + \Big(\frac{\varepsilon^p_{eq}}{\varepsilon_{Y0}}\Big)^n\Big] \quad n \in [0, 1] \tag{68}$$

Ramberg-Osgood:

$$\varepsilon^p_{eq} = \frac{\sigma_Y}{E}\Big[1 + \alpha\Big(\frac{\sigma_Y}{\sigma_{Y0}}\Big)^{m-1}\Big] \quad m \geq 0 \tag{69}$$

8 Equilibrium at large deformations



First Piola-Kirchhoff stress tensor:

$$\mathbf{P} := det(\mathbf{F})\sigma \cdot \mathbf{F}^{-T} = \mathbf{F} \cdot \mathbf{S} \tag{70}$$

Second Piola-Kirchhoff stress tensor:

$$\mathbf{S} := \mathbf{F}^{-1} \cdot \mathbf{P} \tag{71}$$

Kirchhoff stress tensor:

$$\tau = det(\mathbf{F})\sigma = \mathbf{P}\mathbf{F}^T \tag{72}$$

$$div(\mathbf{P}) + \mathbf{b}_0 = 0 \tag{73}$$

Balance of moments:

$$\mathbf{F} \cdot \mathbf{P}^T = (\mathbf{F} \cdot \mathbf{P}^T)^T \tag{74}$$

(Internal) virtual work:

$$\delta W_{int} = \int_V \mathbf{P} : \mathbf{F} dV (= \int_V P_{iA} F_{iA} dV) \tag{75}$$

Internally stored energy + applied/dissipated is smaller than zero:

$$\int_{v_0} \frac{d\psi}{dt} - \mathbf{P} : \dot{\mathbf{F}} dV_0 \leq 0 \quad \Longleftrightarrow \quad \frac{d\psi}{dt} - \mathbf{P} : \dot{\mathbf{F}} \leq 0 \tag{76}$$

8.1 Hyperelastic materials

- Define the kinematic variables $\rightarrow$ what ψ depends on;
- Derive the constitutive restrictions;
- Assign the specific Helmoltz free energy.
- Specify admissibility domain.
- Determine incremental constitutive equations

Kinematic Variables:

$$\psi = \psi(\mathbf{F}) = \psi(\mathbf{C}) \tag{77}$$

Constitutive restrictions:

$$\frac{d\psi}{dt} - \mathbf{P} : \dot{\mathbf{F}} \leq 0 \quad \iff \quad \frac{\partial\psi}{\partial\mathbf{F}} : \dot{\mathbf{F}} - \mathbf{P} : \dot{\mathbf{F}} \leq 0 \quad \iff \quad \Big(\frac{\partial\psi}{\partial\mathbf{F}} - \mathbf{P}\Big) : \dot{\mathbf{F}} \leq 0 \quad \iff \quad \mathbf{P} = \frac{\partial\psi}{\partial\mathbf{F}} = 2\mathbf{F}\frac{\partial\psi}{\partial\mathbf{C}} \tag{78}$$

$$\mathbf{S} = \mathbf{F}^{-1} \cdot \mathbf{P} = \mathbf{F}^{-1} \cdot \Big(2\mathbf{F}\frac{\partial\psi}{\partial\mathbf{C}}\Big) = 2\frac{\partial\psi}{\partial\mathbf{C}} = \frac{\partial\psi}{\partial\mathbf{E}} \tag{79}$$

(since $\mathbf{E} = \frac{1}{2}\Big(\mathbf{C} - \mathbf{I}\Big)$)

$$\tau = \mathbf{P} \cdot \mathbf{F}^T = \mathbf{F} \cdot \frac{\partial\psi}{\partial\mathbf{C}} \cdot \mathbf{F}^T \tag{80}$$

Helmotz free energy: Must satisfy:

- $\psi \rightarrow \infty$, if $det(\mathbf{F}) \rightarrow 0$
- Hooke's law is recovered for small deformations $\sigma = \mathbb{C} : \varepsilon$
- $\psi = 0$ if $\mathbf{F} = \mathbf{I}$

$$\psi(\mathbf{C}) = \frac{1}{2}\mathbf{E} : \mathbb{C} : \mathbf{E} \quad \rightarrow \quad S = \mathbb{C} : \mathbf{E} \tag{81}$$

Saint-Venant

- for isotropic and anisotropic elasticity
- for large rotations and small strain(unphysical under large compression)

$$\mathbf{S} = \lambda tr(\mathbf{E})\mathbf{I} + 2\mu\mathbf{E} \tag{82}$$

Mooney-Rivlin

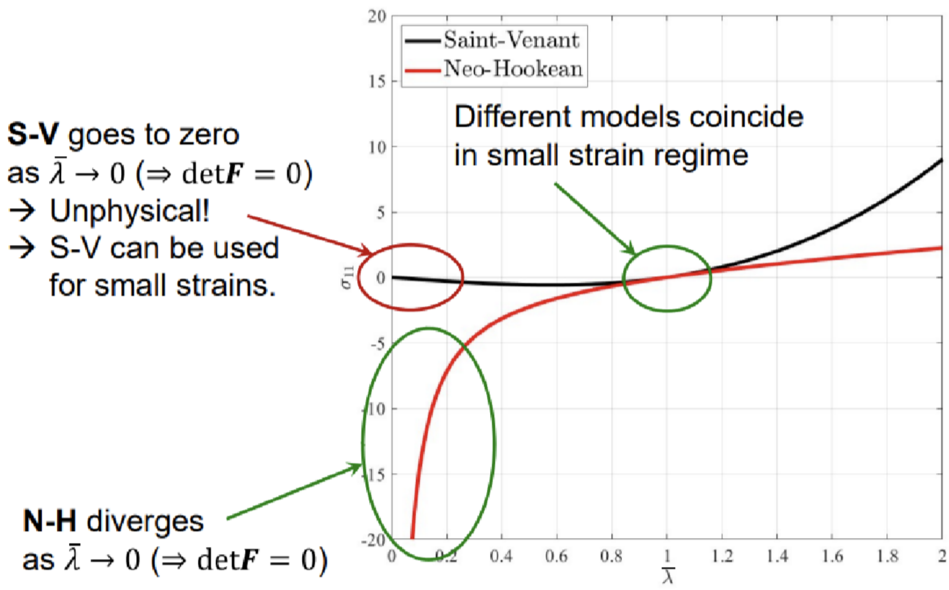
- for isotropic elasticity
- for large strains and rotations
- for **in**compressible materials (rubbers)

$$\mathbf{S} = \Big(\mu_1 - \mu_2 tr(\mathbf{C})\Big)\mathbf{I} + \mu_2\mathbf{C} \tag{83}$$

Neo-Hookean

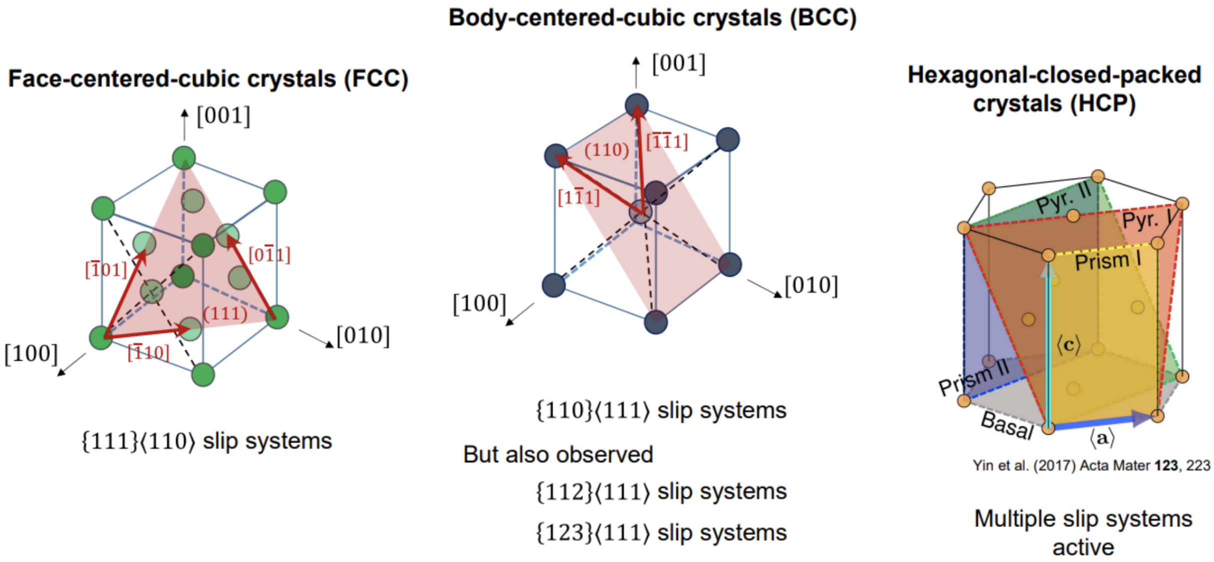
- for isotropic elasticity
- for large strains and rotations
- for compressible materials (rubbers)

$$\mathbf{S} = \mu\Big(\mathbf{I} - \mathbf{C}^{-1}\Big) + \frac{\lambda}{2}\Big(det(\mathbf{F})^2 - 1\Big)\mathbf{C}^{-1} \tag{84}$$



9 Polycrystal plasticity

$[abc]$: specific direction
$< abc >$: family of directions
(abc): specific plane
$\{abc\}$: family of planes



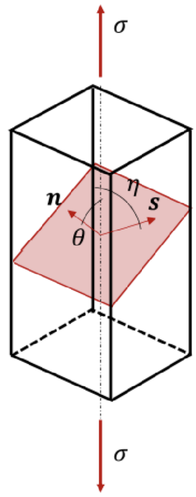


Figure 2: Slip trace due to shear along slip direction

Calculation of shear stress (τ): (*Schmid's law*)

$$\tau = \sigma cos(\theta)cos(\eta) \tag{85}$$

$$\sigma_Y = M \cdot \tau_Y \quad M = 3.067 \tag{86}$$

10 Crystal plasticity

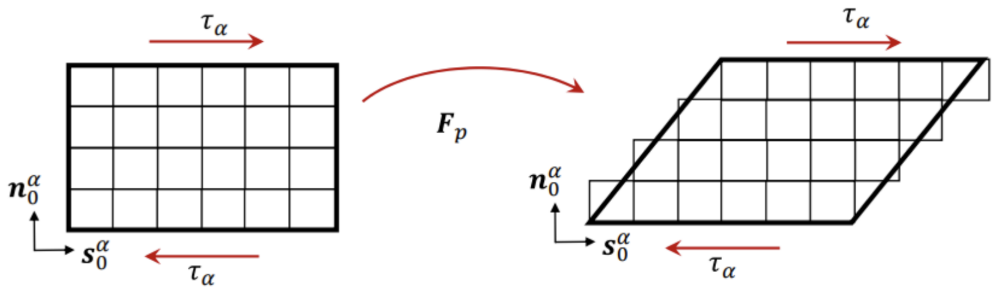


Figure 3: Continuum model

Schmid tensor (shows direction of shear) (slip direction: $\mathbf{s}_0^\alpha$, slip plane normal: $\mathbf{n}_0^\alpha$) :

$$\mathbf{P}_0^\alpha = \mathbf{s}_0^\alpha \otimes \mathbf{n}_0^\alpha \tag{87}$$

Shear stress on α^{th} shear system:

$$\tau_\alpha = \bar{\mathbf{M}} : \mathbf{P}_0^\alpha \tag{88}$$

$\mathbf{L}_p$ shows the direction of plastic deformation:

$$\mathbf{L}_p = \dot{\lambda}\mathbf{R}(\bar{\mathbf{M}}) = \dot{\mathbf{F}}_p \cdot \mathbf{F}_p^{-1} = \sum_{\alpha=1}^{N_s} \dot{\gamma}_\alpha \cdot \mathbf{P}_0^\alpha \tag{89}$$

Plastic velocity gradient:

$$d\dot{\mathbf{l}}_i = \dot{\mathbf{F}}_p \cdot \mathbf{F}_p^{-1} \cdot d\mathbf{l}_i = \mathbf{L}_p \cdot d\mathbf{l}_i \tag{90}$$

Since $\psi = \psi(\mathbf{F}_e) = \psi(\mathbf{F} \cdot \mathbf{F}_p^{-1})$:

$$\mathbf{P} = \frac{\partial \psi}{\partial \mathbf{F}_e} \cdot \mathbf{F}_p^{-T} \tag{91}$$

Dissipation:

$$D = \mathbf{P} : \mathbf{F}_e \cdot \dot{\mathbf{F}}_P = (\mathbf{C}_e \cdot \bar{\mathbf{S}}) : \mathbf{L}_p \geq 0 \tag{92}$$

Rate dependence:

$$\frac{\dot{\gamma}_\alpha}{\dot{\gamma}_0} = \left(\frac{\tau_\alpha}{\tau_{\alpha,Y}}\right)^{\frac{1}{m}} \tag{93}$$

10.1 Hardening

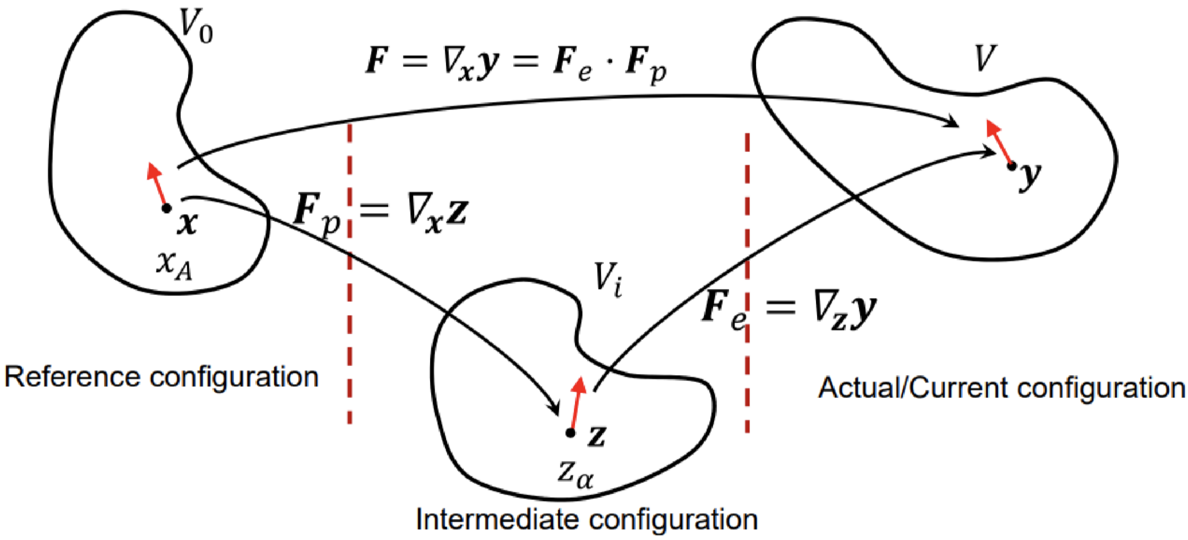
$$\tau_{\alpha,Y} = \tau_{\alpha,0} + \int_0^t \dot{g}_\alpha(\tau) d\tau \tag{94}$$

$$\dot{g}_\alpha(\tau) = h_{\alpha\beta} |\dot{\gamma}_\beta| \tag{95}$$

Hardening matrix:

$$h_{\alpha\beta} = h_0 \left(1 - \frac{\tau_{\beta,\gamma}}{\tau_\infty}\right)^a q_{\alpha\beta} \tag{96}$$

11 Elastoplasticity at large deformations



Total deformation is found using chain rule:

$$\nabla_x \mathbf{Z} = \frac{\partial\, \mathbf{z}(\mathbf{y}(\mathbf{x}))}{\partial \mathbf{y}} \cdot \frac{\partial \mathbf{y}}{\partial x} = \nabla_y \mathbf{Z} \cdot \nabla_x \mathbf{Y} \tag{97}$$

$\mathbf{F}_p(t)$ is history dependent: . $\mathbf{L}_p$ explains in which direction $\mathbf{F}_p$ is changing.
Dissipation:

$$\mathbf{D} = (\mathbf{C}_e \cdot \bar{\mathbf{S}}) : \mathbf{L}_p = \mathbf{P} : \mathbf{F}_e \cdot \dot{\mathbf{F}}_p \geq 0 \tag{98}$$

$$\bar{\mathbf{S}} = \mathbf{F}_e^{-1} \cdot \mathbf{P} \cdot \mathbf{F}_p^T \tag{99}$$

Driving/Mandel stress (acting on intermediate configuration):

$$\bar{\mathbf{M}} = \mathbf{C}_e \cdot \bar{\mathbf{S}} \tag{100}$$

Von Mises equivalent stress:

$$\sigma_{vm} = \sqrt{\frac{3}{2} \bar{\mathbf{M}}^d : \bar{\mathbf{M}}^d} \tag{101}$$

$$f = \bar{M}_{eq}^d - \bar{M}_Y \leq 0 \tag{102}$$

Rate dependent flow:

$$\frac{\dot{\lambda}}{\dot{\lambda}_0} = \left(\frac{\bar{M}_{eq}^d}{\bar{M}_Y} \right)^M \tag{103}$$

Flow direction:

$$\mathbf{R} = \sqrt{\frac{3}{2}} \frac{\bar{\mathbf{M}}^d}{\bar{M}_Y} \tag{104}$$

$$\mathbf{P} = \frac{\partial \psi}{\partial \mathbf{F}_e} \mathbf{F}_p^{-T} \quad \Longleftrightarrow \quad P_{iA} = \frac{\partial \psi}{\partial F_{i\alpha}^e} F_{\alpha A}^{p-1} \tag{105}$$

(if no plasticity, $\mathbf{F}_p = \mathbf{I}$)