Dynamics of Engineering Systems

Name: ____________________

Student number: ____________________

Midterm examination

You have 90 minutes to complete this assignment, individually.

A cube of mass m is connected to a spring of stiffness k and hangs from the ceiling at its equilibrium position (where we define $z = 0$). Assume that no energy is lost in this system (except in section f).

a) Sketch the free-body diagram and write down the equation of motion of the system accounting also for the presence of gravity. (10 pts)
b) Sketch the displacement and velocity time histories for the system on the same plot for the following initial conditions: $z(t_0) = 0$ and $\dot{z}(t_0) = -w$, where w is a positive constant. (10 pts)
c) Sketch the phase plot (displacement versus velocity) for the system. Mark the start of the motion and denote w on the phase plot. (5 pts)
d) Write pseudocode to show how the behavior of the system described in section (a) can be approximated numerically by solving a system of two first-order ODEs (e.g. by calling "ode45"). Make sure to write down explicitly the system of equations to be solved. (15 pts)

The system is now rotated by 90 degrees clockwise so that the mass rests on its side on a rigid surface.

e) Sketch the free-body diagram and the phase plot for the system assuming that friction is zero between the mass and the surface on which it rests, for the following initial conditions: $x(t_0) = -a$ and $\dot{x}(t_0) = 0$. Mark the start of the motion and denote a on the phase plot. (10 pts)
f) Sketch the phase plot for the same system but assuming that friction is non-zero between the mass and the surface on which it rests for the same initial conditions. Mark the start of the motion and denote a on the phase plot. (5 pts)
g) If the stiffness of the spring is $16\pi^2\ N/m$ and the mass of the block is 1 kg, how many times does the center of the block cross the equilibrium position during the first 10 seconds? (Assume, again, that friction is zero in this system.) (15 pts)

Let us now place the horizontally moving frictionless mass-spring system of section (d) on an incline of positive angle θ (measured counterclockwise from the x-axis). Assume that friction is zero in this system.

h) Sketch the free-body diagram of the system in the presence of gravity. (5 pts)
i) Write down the equations of motion for the 2-degree-of-freedom system of section (h). (10 pts)
j) Simplify the system of equations in section (i) to one equation of motion and sketch the corresponding free-body diagram. (Hint: think of a suitable change in coordinate direction and use trigonometry.) (10 pts)
k) Sketch the phase plot of the system in section (j). (5 pts)

Clearly label your plots and make sure to include your name and student number in your solution sheet(s). Be sure to return this sheet with your answers.