Problem 1: Modelling of two-stage problem (20%)

A manufacturer produces 2 products. There are 3 different parts to be ordered from suppliers. A unit of product $i \in \{1, 2\}$ requires a_{ij} units of part $j \in \{1, 2, 3\}$. For example, product 1 requires a_{11} units of part 1, a_{12} units of part 2, and a_{13} units of part 3. The demand for the products is modelled as a random vector $d = (d_1, d_2)$ that has two equally likely scenarios: high demand ($d^h = (d_1^h, d_2^h)$) and low demand ($d^l = (d_1^l, d_2^l)$). Before the demand is known, the manufacturer needs to order the parts at the rate c_j per unit of part $j \in \{1, 2, 3\}$. After the demand d is observed, the manufacturer may decide which portion of the demand is to be satisfied (that is, how many units of each of the product to make), so that the ordered number of parts suffice. In addition to the required parts, it costs additionally l_i to manufacture a unit of product i. The unit selling price of product i is q_i. The parts not used are sold at a salvage value $s_j < c_j$ for all $j \in \{1, 2, 3\}$. The unsatisfied demand is lost. Model this problem as a two-stage stochastic optimization problem (consider all decision variables to take real values and not restricted to integer values). In particular, do the following:

1. Determine the first-stage decision variables, costs, and constraints. **[Score 6%]**
2. Determine the second-stage decision variables, costs, and constraints. **[Score 6%]**
3. Using the above two, formulate the two-stage recourse problem. **[Score 5%]**
4. Does the problem admit relatively complete recourse (justify your answer)? **[Score 3%]**

Problem 2: Convex analysis (7%)

Determine if the following set $\mathcal{F}$ is convex or not. Justify your answer with appropriate reasons.

$$\mathcal{F} = \{x \mid Ax \geq b_1\} \cup \{x \mid Ax \geq b_2\},$$

where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}, \; b_1 = \begin{bmatrix} 0 \\ 0 \\ -3 \\ -1 \end{bmatrix}, \; \text{and } b_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -3 \end{bmatrix}.$$

Problem 3: Probability (10%)

Let the distribution of the random variable ξ be given as

$$\mathbb{P}(\xi = i) = \frac{c * i}{5}, \qquad \text{for } i = 1, 2, \ldots, 5,$$

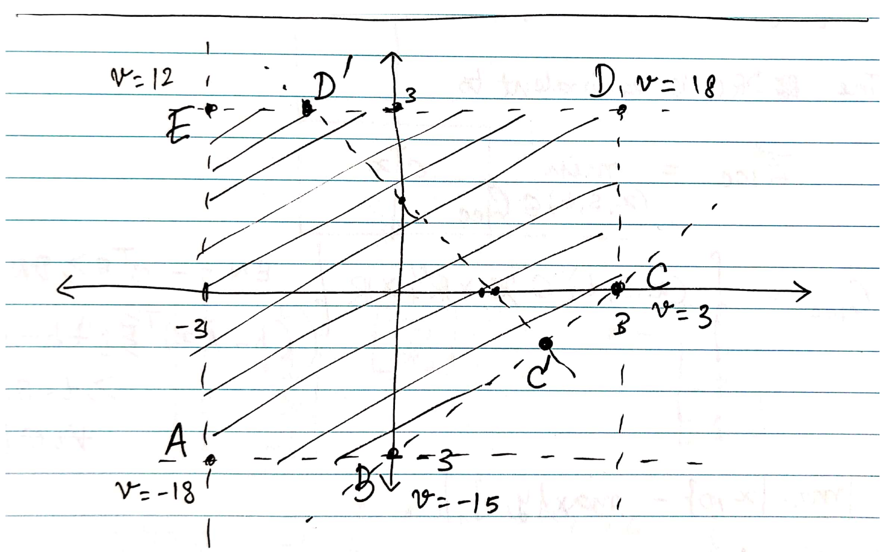
1. Find the value of c that makes the above distribution valid. **[Score 5%]**
2. Compute the 0.6 quantile of the distribution. **[Score 5%]**

Problem 4: Linear programming (8%)

Find the solution to the following linear program **[Score 6%]**

$$\begin{aligned} \min \quad & x_1 + 5x_2 \\ \text{subject to} \quad & x_1 - x_2 \leq 3, \\ & -3 \leq x_1 \leq 3, \\ & -3 \leq x_2 \leq 3. \end{aligned}$$

Does the solution change when you add another constraint to the above problem: $x_1 + x_2 \leq 1$ (justify your answer)? **[Score 2%]**



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Problem 5: Properties of two-stage problems (20%)

Consider the second-stage problem

$$\begin{aligned} \min \quad & y_1 + y_2 \\ \text{subject to} \quad & y_1 + y_2 \geq 2x_2 - \xi_1, \\ & 0 \leq y_1 \leq 3, \\ & 0 \leq y_2 \leq x_1 + \xi_2. \end{aligned}$$

Do the following:

1. For any $\xi = (\xi_1, \xi_2)$, compute $K_2(\xi)$ (see the footnote[1]). **[Score 12%]**
2. Let $\xi = (\xi_1, \xi_2)$ take these four values with equal probability

$$\{(0.2, 0.3), (0, 0.5), (0.5, 0), (1, 1)\}.$$

   Then, compute K_2 (see the footnote[2]). **[Score 8%]**

[1] Recall that this set is the set of first-stage decisions that ensure feasibility in the second stage for the particular uncertainty ξ.

[2] Recall that this is the set of first-stage decisions that ensure feasibility in the second stage for all realizations of the uncertainty ξ.

Problem 6: Computing EVPI and VSS (25%)

Consider the following two-stage problem

$$\begin{aligned} \min \quad & x + 5\mathbb{E}_{\xi}[|x - \xi|] \\ \text{subject to} \quad & 0 \leq x \leq 3. \end{aligned}$$

Let the random variable take two values $\xi_1 = 1$ and $\xi_2 = 5$. Let the probability distribution be given as

$$p_1 = \mathbb{P}[\xi = \xi_1] = 0.4, \text{ and } p_2 = \mathbb{P}[\xi = \xi_2] = 0.6.$$

Do the following

1. Determine the value of the wait-and-see solution (WS). **[Score 7%]**
2. Determine the value of the recourse problem (RP). **[Score 7%]**
3. Determine the expected result of using the expected value solution (EEV). **[Score 7%]**
4. Using the above values, determine the expected value of perfect information (EVPI) and the value of stochastic solution (VSS). **[Score 4%]**