Student name (surname and name):

Student ID:

Before the examination:

- The duration of the exam is 2 hours.
- Each problem page is followed by blank pages. If you cannot fit your solutions there, ask for extra sheets.
- Your solutions must be in English.
- The exam has four problems and the total number of points is 70.

During the examination:

- You are allowed to use the text book (Birge and Louveaux), lecture slides, tutorial material, practice problems document, and any other hand written notes.
- You are not allowed to use any electronic aids (such as laptops, calculators, cell phones, etc.) and any other book or print outs.
- You must use a pen (blue or black) to write your solutions. Do not write with a pencil.
- Do not communicate with your colleagues.
- Only solutions written in the solution booklet and in the provided extra sheets will be graded.
- Justify your solutions. Correct final results will receive full credit only if the way in which the result was reached is documented.
- If you return the sheet, then your exam will be graded, unless you explicitly write "do not grade". Even if you write "do not grade" please write your Surname, Name, Student ID on the first page.
- If you want to finish before time, submit the answer sheet and any additional sheets (if present) and leave the room.

At the end of the examination:

- Stop writing and remain seated until instructed that you are free to leave.
- Make sure your answer sheet is collected.

Problem	1	2	3	4	Total
Score					

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Problem 1: Probability theory (14 Points)

Part 1 (10 points) Let X be a random variable with expected value and variance given as $\mathbb{E}[X] = 2$ and $\text{Var}[X] = 1$. Calculate the expected value of the new random variable $Y = (2X + 1)^2$.

Part 2 (4 points) Let X be a random variable with distribution

$$\mathbb{P}[X = i] = \frac{c}{i^2}$$

for $i = 1, 2, 3, \ldots$ with c being an appropriate positive constant that makes the distribution valid. Is the $\text{CVaR}_{1-p}(X)$ with $p = 0.5$ finite or not? Justify your answer.

Problem 2: Convex analysis (16 Points)

Each part is worth 8 points.

Part 1: Compute the subdifferential of a function $g(x) := \max\{x - 5, -3x + 15, 2x - 10\}$ at the point $x = 5$. That is, compute $\partial g(5)$. Hint [1]

Part 2: Let $X \subset \mathbb{R}^n$ be a convex set. Let $A \in \mathbb{R}^{m\times n}$ be a $m \times n$ matrix and $b \in \mathbb{R}^m$ be a vector. Consider another set $Y \subset \mathbb{R}^n$ defined as

$$Y = AX + b := \{Ax + b \mid x \in X\}.$$

Is Y convex? Justify your answer.

[1] It might be helpful to plot the function g and understand how it behaves.

Problem 3: Properties of two-stage programs (20 Points)

Consider the second-stage problem where x is the first-stage variable and y is the second-stage variable

$$\begin{aligned} \min \quad & 3y_1 + 4y_2 \\ \text{subject to} \quad & y_1 + y_2 \geq \xi x_1 - x_2, \\ & y_1 + 2y_2 \leq x_1 + \xi x_2, \\ & y_1 \geq 0, y_2 \geq 0. \end{aligned}$$

Do the following:

1. For any ξ, compute $K_2(\xi)$ (see the footnote[2]). **[12 points]**
2. Let ξ take two values with equal probability

$$\{0.2, 0.6\}.$$

Then, compute K_2 and plot it in a 2-dimensional plot(see the footnote[3]). **[8 points]**

[2] Recall that this set is the set of first-stage decisions that ensure feasibility in the second stage for the particular uncertainty ξ.

[3] Recall that this is the set of first-stage decisions that ensure feasibility in the second stage for all realizations of the uncertainty ξ.

Problem 4: EVPI and VSS (20 Points)

Consider the following two-stage problem

$$\begin{array}{rl} \min & 2x + \mathbb{E}_{\xi}[|x-\xi| + |2x-\xi|] \\ \text{subject to} & 0 \leq x \leq 4. \end{array}$$

Let the random variable take two values $\xi_1 = 1$ and $\xi_2 = 10$. Let the probability distribution be given as

$$p_1 = \mathbb{P}[\xi = \xi_1] = 0.5, \text{ and } p_2 = \mathbb{P}[\xi = \xi_2] = 0.5.$$

Do the following

1. Determine the value of the wait-and-see solution (WS). **[6 points]**
2. Determine the value of the recourse problem (RP). **[6 points]**
3. Determine the expected result of using the expected value solution (EEV). **[6 points]**
4. Using the above values, determine the expected value of perfect information (EVPI) and the value of stochastic solution (VSS). **[2 points]**