Dynamics of Engineering Systems Name: _____

Student number: _____

Final examination

You have 90 minutes to complete this assignment, individually.

A viscoelastic (polymeric) cylindrical rod of diameter D and length L is initially at a glassy state (at low temperature). Assume that the Maxwell model can adequately represent the behavior of this polymer.

a) Sketch the stress-strain behavior as well as the stress as a function of time, for the case when we apply a small finite strain ε_0 on the rod between times t_0 and t_f. (10 pts)

We now bring one of the circular faces of the rod into full contact with a heated plane (i.e. the plane has temperature that is higher than the temperature of the rod, so $T_p > T_r$). A constant heat flux $\overrightarrow{q_s}$ is present at the contact. Radiation is negligible.

b) Sketch the temperature distribution in the rod (i.e., its temperature in the axial direction $x \in [0, L]$) at different times $t \in [t_0, t_1, t_2, t_\infty]$. Specify which face of the rod is in full contact with the heated plane on your plot. (10 pts)

c) Given the general conservation equation $\frac{\partial q}{\partial t} + \nabla \cdot \vec{F} = P_v$ and Fourier's law $\vec{q} = -k\nabla T$, derive the equation relating the time-evolution of temperature in the rod to the distribution of temperature along the axial direction given that the first term in the general conservation equation is $\frac{\partial q}{\partial t} = \rho C_p \frac{\partial T}{\partial t}$. (10 pts)

d) Write down the initial and boundary conditions for the system described in parts b) and c). (10 pts)

e) Write down pseudocode to describe how the system described in parts b), c) and d) can be solved numerically if the heat transfer rate is $\dot{Q}_s$. Be sure to include the discretized equations to be solved, as well as the initial and boundary conditions. (30 pts)

Now that the temperature in the rod has increased, the polymer is no longer in a glassy state.

f) Sketch the output of a creep test on the rod and describe what happens in the plot relative to the definition of a Maxwell model. (10 pts)

g) Sketch the response of the rod to a stress impulse for this Maxwell model (i.e. stress that is applied and released almost instantaneously). Explain your reasoning for this behavior. (10 pts)

Clearly label your plots and make sure to include your name and student number in your solution sheet(s).

Be sure to return this sheet with your answers.