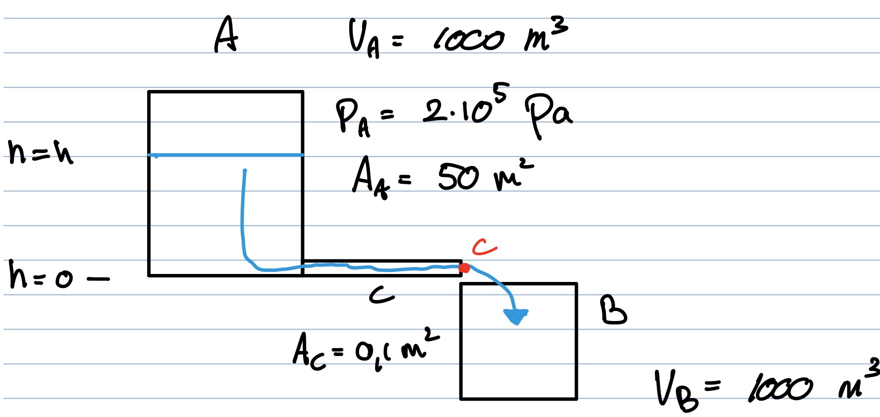
Sample answers - Lugus Fluid Dynamics Exam Support

question 1



$$\frac{P_A}{\rho} + \cancel{\frac{v_A^2}{2}} + gh_A = \cancel{\frac{P_c}{\rho}} + \frac{v_c^2}{2} + \cancel{gh_c}$$

point c just outside the pipe, atmospheric pressure

$A_A >> A_c$ so negligible v_A

$$V_c = \sqrt{2gh_A + 2\frac{Pa}{\rho}} = \sqrt{2} \cdot \sqrt{gh_A + \frac{P_a}{\rho}}$$

$$V_c = \sqrt{2\frac{P_a - P_c}{\rho} + 2gh_a}$$

(if you don't take P_c as BC)

Mass balance

$$\frac{d\,x V}{dt} = \frac{d\,\rho \cdot V}{dt} = \cancel{in} - out + \cancel{prod}$$

$$\cancel{\rho}\frac{dV}{dt} = -\phi_{v\,out} \cdot \cancel{\rho}$$

$$\phi_{V_{out}} = A_c \cdot V_c$$

$$V = A_A \cdot h$$

at $t=0$; $h = \frac{V}{A_A}$

$h = \frac{1000}{50} = 20\ m$

at $t=t$; $h=0$

BC's

$$\frac{dV}{dt} = \frac{d A_A \cdot h}{dt} = A_A \frac{dh}{dt}$$

$$A_A \cdot \frac{dh}{dt} = -A_c \cdot \sqrt{2} \cdot \sqrt{g h_a + \frac{P_a}{\rho}}$$

$$\int_{20}^{0} \frac{1}{\sqrt{g h_a + \frac{P_a}{\rho}}} dh = -\frac{A_c}{A_A} \cdot \sqrt{2} \cdot \int_0^t dt$$

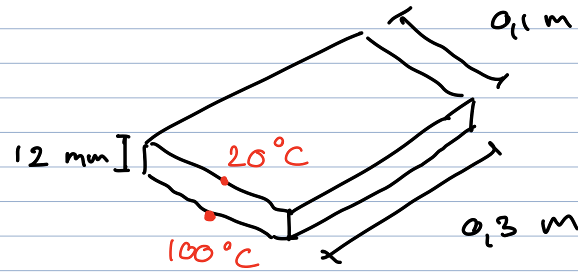
$$\int \frac{1}{\sqrt{ax+b}} dx = \frac{2}{a}\sqrt{ax+b}$$

$$\left[\frac{2}{g}\sqrt{g h_a + \frac{P_a}{\rho}}\right]_{20}^{0} = 0 - \frac{2}{10} \cdot \sqrt{200 + \frac{2 \cdot 10^5}{10^3}} = -4$$

$$-4 = -\frac{A_c}{A_A} \cdot \sqrt{2} \cdot t$$

$$t = \frac{4}{\sqrt{2}} \cdot \frac{A_A}{A_c} = 1414 \text{ seconds}$$

question 2



$A_{slab} = 0{,}3 \cdot 0{,}1 = 0{,}03 \ m^2$

Stainless steel , $\lambda = 16 \ W/mK$

$$\phi_q'' = h \cdot (T_1 - T_0)$$

stationary heat conduction in flat plane

(temperature in slab steady state w.r.t time)

$$\phi_q'' = \lambda \cdot \frac{T_1 - T_0}{d}$$

See lecture 6 slide 5

$$h \cdot (T_1 - T_0) = \lambda \frac{(T_1 - T_0)}{d}$$

$$h = \frac{\lambda}{d} = \frac{16}{0{,}012} = 1333{,}\overline{33} \ \frac{W}{m^2 \cdot K}$$

Question 3

$V_{real} = 1{,}25 \ m^3$

$V_{model} = 0{,}01 \ m^3$

$P_o = f(N, \rho, \eta, D)$

Ideal conditions (model) $\rightarrow$ $N = 150$ RPM , $P_o = 70 W$

a) Units

$P_o = [W] = \left[\frac{J}{s}\right] = \left[\frac{N \cdot m}{s}\right] = \left[\frac{kg \cdot m \cdot s^{-2} \cdot m}{s}\right] = \left[\frac{kg \cdot m^2}{s^3}\right]$

$N = [s^{-1}]$

$\rho = \left[\frac{kg}{m^3}\right]$

$\eta = [Pa \cdot s] = [N \cdot m^{-2} \cdot s] = [kg \cdot m \cdot s^{-2} \cdot m^{-2} \cdot s] = \left[\frac{kg}{m \cdot s}\right]$

$D = [m]$

Buckingham's π theorg :

Dimless numbers = variables - dimensions

Dimless numbers = $5 - 3 = 2$

b) $P_0 = N^a \cdot \rho^b \cdot \eta^c \cdot D^d$

$$\frac{kg \cdot m^2}{s^3} = \frac{1}{s}^a \cdot \frac{kg}{m^3}^b \cdot \frac{kg}{m \cdot s}^c \cdot m^d$$

kg : $1 = b + c$	$b = 1 - c$
m : $2 = -3b - c + d$	$a = 3 - c$
s : $-3 = -a - c$	$2 = -3 + 3c - c + d$
	$d = 5 - 2c$

$$P_0 = N^{3-c} \cdot \rho^{1-c} \cdot \eta^c \cdot D^{5-2c}$$

$$\frac{P_0}{N^3 \cdot \rho \cdot D^5} = \left(\frac{\eta}{N \cdot \rho \cdot D^2}\right)^c$$

c)

$$\left(\frac{\eta}{N \cdot \rho \cdot D^2}\right)^c \Bigg|_{model} = \left(\frac{\eta}{N \cdot \rho \cdot D^2}\right)^c \Bigg|_{real}$$

Viscosity and density remain the same

$$\left(\frac{\cancel{\eta}}{N\cdot\cancel{\rho}\cdot D^2}\right)^c\Bigg|_{model} = \left(\frac{\cancel{\eta}}{N\cdot\cancel{\rho}\cdot D^2}\right)^c\Bigg|_{real}$$

$$\frac{1}{N_m\cdot D_m^2} = \frac{1}{N_r\cdot D_r^2}$$

$$N_r = \frac{N_m\cdot D_m^2}{D_r^2} = N_m\cdot\left(\frac{D_m}{D_r}\right)^2$$

Also, same dimensions

$$\frac{V_{model}}{V_{real}} = \frac{0{,}01}{1{,}25} = 0{,}008$$

$$\frac{L}{D}\Bigg|_{mod} = \frac{L}{D}\Bigg|_{real}$$

$$L_m = \frac{L_r\cdot D_m}{D_r} = L_r\cdot\frac{D_m}{D_r}$$

$$\frac{\pi}{4}D_m^2\cdot L_m = \frac{\pi}{4}D_r^2\cdot L_r\cdot 0{,}008$$

$$\cancel{\frac{\pi}{4}}D_m^2\cdot\cancel{L_r}\cdot\frac{D_m}{D_r} = \cancel{\frac{\pi}{4}}D_r^2\cdot\cancel{L_r}\cdot 0{,}008$$

$$\frac{Dm^3}{Dr} = \frac{Dr^2}{0{,}008}$$

$$Dm^3 = 0{,}008\ Dr^3$$

$$125\ Dm^3 = Dr^3$$

$$Dr = 5Dm$$

$$Nr = Nm \cdot \left(\frac{Dm}{Dr}\right)^2 = 150 \cdot \left(\frac{Dm}{5Dm}\right)^2 = 6\ \text{RPM}$$

d) $$\left.\frac{P_o}{N^3 \cdot \cancel{\rho} \cdot D^5}\right|_{model} = \left.\frac{P_o}{N^3 \cdot \cancel{\rho} \cdot D^5}\right|_{real}$$

$$\frac{P_{o_m}}{N_m^3 \cdot D_m^5} = \frac{P_{o_r}}{N_r^3 \cdot D_r^5}$$

$$P_{o_r} = \frac{P_{o_m} \cdot N_r^3 \cdot D_r^5}{N_m^3 \cdot D_m^5} = P_{o_m} \cdot \left(\frac{N_r}{N_m}\right)^3 \cdot \left(\frac{D_r}{D_m}\right)^5$$

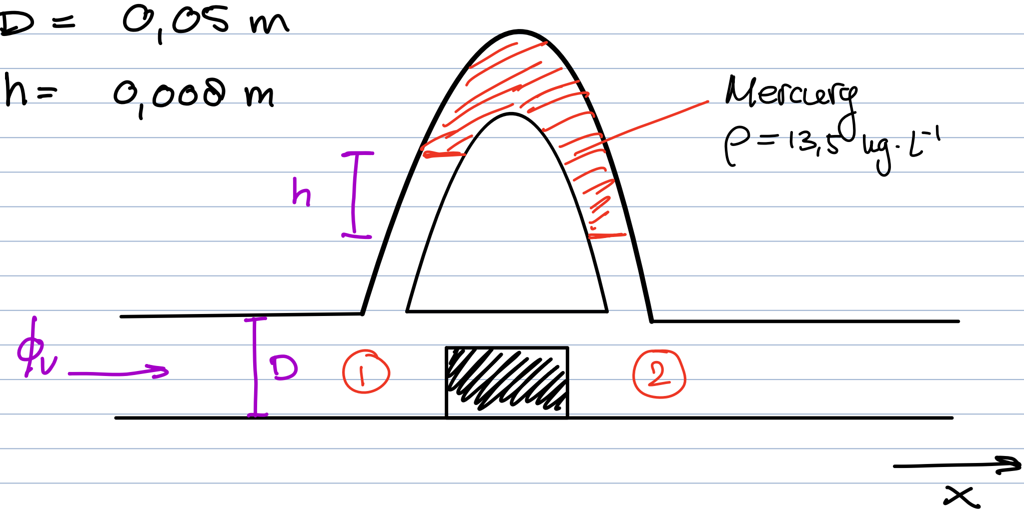
$$P_{o_r} = 70 \cdot \left(\frac{6}{150}\right)^3 \cdot \left(\frac{5Dm}{Dm}\right)^5 = 14\ W$$

Question 4

$\phi_v = 2\ L/s$

$D = 0{,}05\ m$

$h = 0{,}008\ m$



$$V \cdot \rho \cdot \frac{dv}{dt} = \phi_v \cdot \rho \cdot v_{in} - \phi_v \cdot \rho \cdot v_{out} + \Sigma F_x$$

Assume steady state

$$0 = \phi_v \cdot \rho \cdot (V_{in} - V_{out}) + \Sigma F_x$$

Constant diameter → constant flow velocity

$V_{in} = V_{out}$

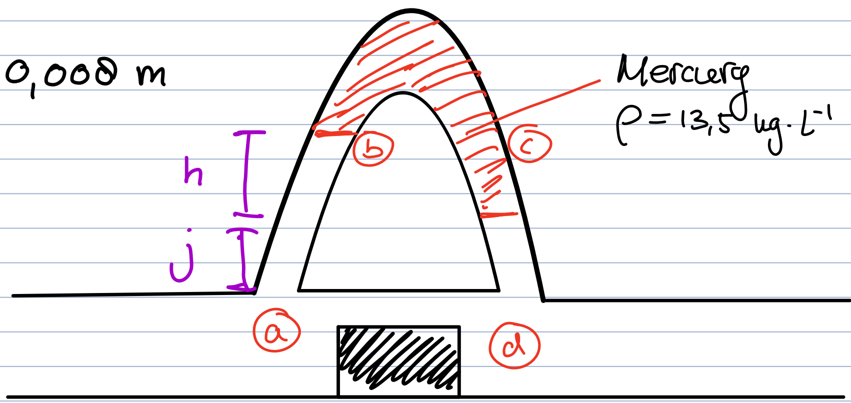
$$0 = \Sigma F_x$$

$$\Sigma F_x = P_1 \cdot A_1 - P_2 \cdot A_2 + F_b$$

$$F_b = -A \cdot (P_1 - P_2) = A \cdot (P_2 - P_1)$$

Assume simplified Bernouilli holds
Calculate pressures from bend tube

h = 0,008 m



$$\frac{P_a}{\rho} + \cancel{\frac{V_a^2}{2}} + \cancel{g h_a} = \frac{P_a}{\rho} + \cancel{\frac{V_a^2}{2}} + g h_a$$

$$\frac{P_a - P_b}{\rho} = g h_b \qquad P_a - P_b = \rho g h$$

$$P_a - P_b = 1000 \cdot 10 \cdot (h+j) = 10000(h+j)$$

$$P_c - P_d = \rho g h \big|_{water} + \rho g h \big|_{mercury}$$

$$P_c - P_d = h \cdot 10 \cdot 13500 + j \cdot 10 \cdot 1000$$

$P_b - P_c = 0$ (same hight position)

$$P_a - P_b = 10000(h+j)$$
$$P_b - P_c = 0$$
$$P_c - P_d = 135000h + 10000j$$

$$P_a - P_d = 10000(h+j) - 10000j - 135000h$$

$$P_a - P_d = 10000h - 135000h$$

$$P_a - P_d = -1000 \text{ Pa}$$

$$F_b = A \cdot (P_2 - P_1)$$

$$F_b = \pi r^2 \cdot (P_d - P_a) = \pi \cdot 0{,}025^2 \cdot (1000) = 1{,}96 \text{ N}$$