# Advanced Digital & Hybrid Control Systems – A.Y. 20/21

## 1 Instructions

- The assignment determines your grade, and you are supposed to work on it **by yourself**. It is mandatory that you **fill in and sign** the declaration also attached to the email.
- Solve **either** Questions 1a and 2 **or** Questions 1b and 2. The former combination (1a+2) is more standard and corresponds to maximum 8 points, the latter combination (1b+2) requires a deeper understanding and corresponds to maximum 10 points. Each subquestion contains an **indication** of the number X of its points as "∼X points".
- You **are** allowed to reuse the MATLAB/Simulink code from tutorials and practicals.
- You need to return a compressed folder (e.g., a zip folder) with the following.

  1. The signed declaration that was attached to the email.
  2. The report with your answers to the questions in the assignment as a **pdf** file. Name the file "assignment_`X`.pdf" where `X` stands for your surname.
  3. A **single** MATLAB script **for each** question with all the code that you needed to obtain the numerical results and the figures of the report. Name each file "script_`X`_`Y`.m" where `X` stands for your surname and `Y` for the question number.
  4. The Simulink files that are called by the previous MATLAB script.

  The self-contained written report is what counts and what I will check. However, I reserve the possibility of **running** the MATLAB and Simulink files in case something is not clear to me from the written report.
- In case you are going to write the report in LaTeX and would like the tex source of this pdf, I am happy to provide it to you.
- **Evaluation criteria** for the written report.

  For all questions, you need to justify your answer: mention which results from the lectures you invoke and write down all the derivations that are necessary to support the answer. In descending order of importance, your answers should be **convincing**, **clear** and **concise**.

  For all figures, indicate x- and y-labels and select suitably the limits of the time axis so that the transients of the time responses can be appreciated.
- Email to `a.bisoffi@rug.nl` (with subject `[ADHCS 21 - exam]`) the aforementioned compressed folder **before April 12, 2020 23:59 hrs** (Central European Summer Time). This is witnessed by the time I receive the email.
- Good luck!

## 2 Assignment

### 2.1 Steering of a cargo ship

Autopilots for ships are typically based on feedback from a heading measurement, which is provided by a gyrocompass, to a steering engine, which drives the rudder [R1]. The (nonlinear) ship dynamics are obtained by Newton's equations. It is customary to describe the straight-line course of the ship by a coordinate system fixed to the ship, and the relevant physical quantities are the magnitude of the forward velocity, the angular velocity, the heading angle and the rudder angle, which are denoted respectively by $\phi$, $r$, $\psi$ and $u$ as in Figure 1. We consider $x = \begin{bmatrix} r \\ \psi \end{bmatrix} \in \mathbb{R}^2$ as the state and $u \in \mathbb{R}$ as the input. We linearize the equations of motion around the steady state $\phi = \phi_0$ and $r = \psi = u = 0$ and consider two principal conditions of motion, namely at nominal speed (condition 1) and of deceleration (condition 2). For these two conditions the dynamics read

$$\dot{x} = \mathcal{A}_q x + \mathcal{B}_q u$$

with $q \in \{1, 2\}$ and

$$\mathcal{A}_q = \begin{bmatrix} -3\frac{\phi_q}{\phi_0} & 0 \\ 1 & 0 \end{bmatrix}, \quad \mathcal{B}_q = \begin{bmatrix} 8\left(\frac{\phi_q}{\phi_0}\right)^2 \\ 0 \end{bmatrix},$$

and $\phi_1 = \phi_0$ for nominal speed and $\phi_2 = 0.17\phi_0$ for deceleration.

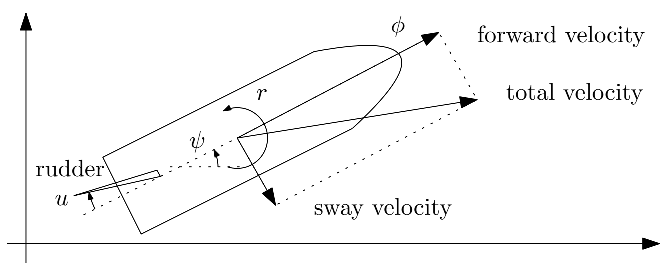



Figure 1: Physical variables.

While referring to this common introduction, solve now either Question 1a or Question 1b.

**Question 1a. (5 points)**

The objective of this question is to analyze the switching system

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t)$$

with switching signal $\sigma : \mathbb{R}_{\geq 0} \to \{1, 2\}$; $A_1 = \mathcal{A}_1$, $B_1 = \mathcal{B}_1$; $A_2 = \mathcal{A}_2$, $B_2 = \mathcal{B}_2$. Consider the case of $y = \psi \in \mathbb{R}$ as output. To compensate for different ship conditions, a switching controller

$$u(t) = k_{\sigma(t)}y(t)$$

is considered with $k_1 = -3$ and $k_2 = -6$.

- **($\sim$1 point)** Derive the equations of the closed-loop system as

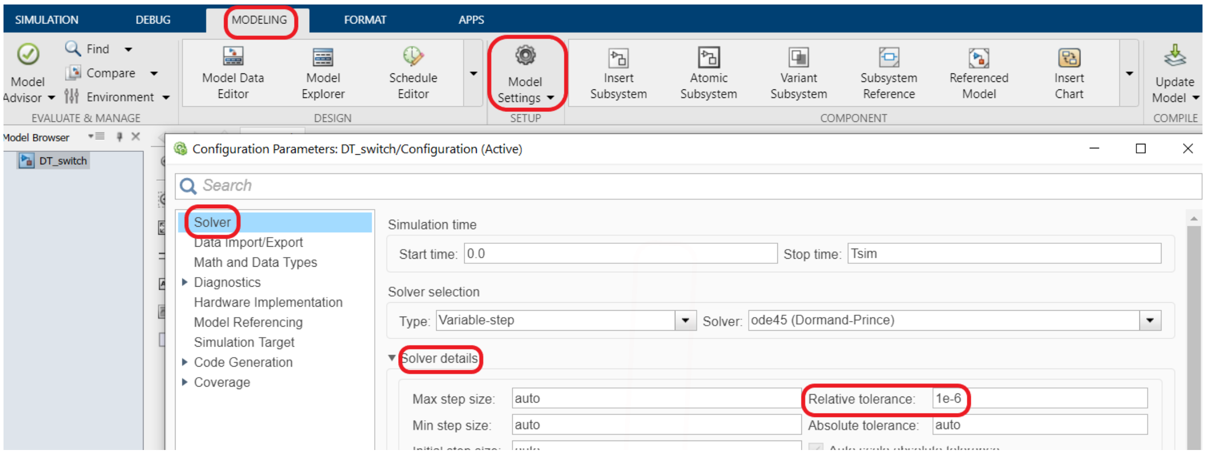
$$\dot{x}(t) = \Phi_{\sigma(t)}x(t) \tag{1}$$

and determine which of the matrices $\Phi_1$, $\Phi_2$ is stable.

- **($\sim$2.5 points)** Find a value for $\tau_D$ such that (1) is guaranteed to be GUAS with respect to $\mathcal{S}_{\text{dwell}}[\tau_D]$. For this $\tau_D$, is (1) GUAS with respect to $\mathcal{S}_{\text{ave}}[\tau_D, 10]$? Justify why it is or why it is not.

  In case one of the two matrices $\Phi_1$ or $\Phi_2$ is not stable, find constraints on the activation time of the unstable modes under which the GUAS property can be achieved.

- **($\sim$1.5 points)** Create a MATLAB/Simulink file to simulate the evolution of (1). For better accuracy, set the parameter "Relative tolerance" to $10^{-6}$ as in the picture below for MATLAB R2019b.




  Then, simulate the evolution of (1) ensuring that: i) the initial condition is $x(0) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$; and ii) the switching signal $\sigma$ belongs to $\mathcal{S}_{\text{dwell}}[1]$. Plot the time evolution of $x$, $u$ and $\sigma$.

**Question 1b. (7 points)**

The objective of this question is to obtain a simplified multi-model adaptive switching control based on Lecture 8 for when the ship is decelerating (condition 2). Consider then for the ship the dynamics

$$\dot{x} = A_{p^\star} x + B_{p^\star} u \text{ with } A_{p^\star} = \mathcal{A}_2 \text{ and } B_{p^\star} = \mathcal{B}_2.$$

For simplicity, consider the case when the output $y = C_{p^\star} x$ coincides with the state $x$. However, as in Lecture 8, the controller does not know which dynamics the ship is following, but only that the dynamics can be given either by matrices $A_1 = \mathcal{A}_1$, $B_1 = \mathcal{B}_1$, $C_1 = I$ or $A_2 = \mathcal{A}_2$, $B_2 = \mathcal{B}_2$, $C_2 = I$ (where $I$ denotes an identity matrix); these matrices form then the family

$$\{(A_p, B_p, C_p) \colon p \in \{1, 2\}\}.$$

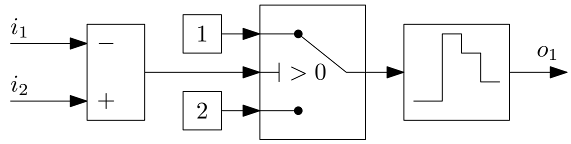
- **($\sim$1 point)** Design the controllers $K_1$ and $K_2$ such that $A_1 + B_1 K_1$ and $A_2 + B_2 K_2$ have eigenvalues, respectively, in $-5 \pm j0.1$ and $-1 \pm j0.02$, where $j$ is the imaginary unit.

  Design the gains $L_1$ and $L_2$ of the asymptotic observers such that both $A_1 + L_1 C_1$ and $A_2 + L_2 C_2$ have eigenvalues in $-10$ and $-8$.

- **($\sim$1 point)** The dwell-time switching times are given by $t_k = k\tau_D$ with $k = 0, 1, 2, \ldots$. Determine the parameter $\tau_D$ to guarantee asymptotic stability of the scheme. In the relevant Lyapunov equations, select $Q_1 = Q_2 = I$.

- **($\sim$4 points)** Implement the scheme in Simulink, and comment it in the report by specifying how the portions of the multi-model adaptive switching control scheme and their equations correspond to which Simulink blocks.

*Hint: You may find useful the set of blocks below. If you use it in your Simulink scheme, justify in the report what it achieves.*




*In this set of blocks, the right-most one is a zero-order hold where a sample time $T_s$ was specified in the "Block Parameters" so that the block samples its input at instants $t_\ell = \ell T_s$, $\ell = 0, 1, 2, \ldots$. For inputs $i_1$ and $i_2$, the generated output $o_1$ is*

$$\text{for all } \ell = 0, 1, 2, \ldots \text{ and for all } t \in [t_\ell, t_{\ell+1}), \quad o_1(t) = \begin{cases} 1 & \text{if } i_1(t_\ell) < i_2(t_\ell) \\ 2 & \text{if } i_2(t_\ell) \leq i_1(t_\ell). \end{cases}$$

- **($\sim$1 point)** Consider initial conditions $\left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right]$ for the process and $\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$ for both asymptotic observers. Plot the time evolutions of $x$, $u$, and of the switching signal $\sigma$.

## 2.2 Event-triggered control of a platoon of trucks

Platoons of trucks are becoming popular since keeping a close formation helps reduce the air drag and increase fuel efficiency. We consider here a platoon of two trucks and we are interested in achieving asymptotic stability of a steady state condition through an event-based controller, so that vehicle-to-vehicle communications are reduced. The steady-state condition corresponds to a desired distance $\bar{d}$ between the two trucks and a desired cruise velocity $\bar{v}$ for the platoon. The relevant physical quantities are $s_i$, $v_i$ and $a_i$, which are respectively absolute position, velocity and acceleration of truck $i$, $i \in \{1, 2\}$. Then, the state variable $x$ and input variable $u$ are

$$x = \begin{bmatrix} s_1 - s_2 - \bar{d} \\ v_1 - \bar{v} \\ v_2 - \bar{v} \end{bmatrix} \in \mathbb{R}^3 \text{ and } u = \begin{bmatrix} a_1 - \gamma_1 \bar{v} \\ a_2 - \gamma_2 \bar{v} \end{bmatrix} \in \mathbb{R}^2$$

where $\gamma_1$ and $\gamma_2$ are normalized viscous friction coefficients. The values of the parameters are $\bar{d} = 8$ m, $\bar{v} = 8.5$ m/s, $\gamma_1 = 0.01$ s$^{-1}$, $\gamma_2 = 0.005$ s$^{-1}$. The relevant dynamics of the platoon read

$$\dot{x}(t) = Ax(t) + Bu(t) = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -\gamma_1 & 0 \\ 0 & 0 & -\gamma_2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

and is complemented by control

$$u(t) = Kx(t_k) = \begin{bmatrix} -0.015 & -0.29 & 0.10 \\ 0.015 & 0.10 & -0.29 \end{bmatrix} x(t_k)$$

with $t_k$ ($k = 0, 1, 2, \ldots$) being a generic sampling time. Select the design parameter $\sigma$ of the scheme as

$$\sigma = \frac{1}{2} \cdot \frac{\lambda_{\min}(Q)}{2\|PBK\|}$$

with the quantities $P$ and $Q$ defined as in Lecture 3, and in particular $Q = I$, where $I$ is an identity matrix.

**Question 2. (3 points)**

- **(∼1 point)** Implement an event-based logic [R2] to guarantee asymptotic stability of the aforementioned steady-state condition through a MATLAB/Simulink file.
- **(∼1 point)** Simulate the evolution of the closed-loop system for initial condition $x(0) = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$ and present the figures with the time evolution of $x$, of $u$ and of the inter-sampling times.
- **(∼1 point)** To illustrate the bound $|e| \leq \sigma|x|$, with $|e|$ and $\sigma$ defined as in Lecture 3, create a figure with $\frac{1}{\sigma}|e|$ and $|x|$. With $\exp(\cdot)$ being the natural exponential function, derive an upperbound of the form

  $$|x(t)| \leq c|x(0)| \exp(-\lambda t), \quad t \geq 0 \tag{2}$$

  and compute numerically the values of $c$ and $\lambda$ for the platoon. Plot then $|x|$ and its upperbound in (2) in a different figure and comment it.

  *Hint: To derive the upperbound, use the reasoning we saw in* Part 2 *of the course.*

**References.**


[R1] K.J. Åström and B. Wittenmark. Adaptive Control. Prentice Hall, 1994

[R2] P. Tabuada. Event-Triggered Real-Time Scheduling of Stabilizing Control Tasks. IEEE Transactions on Automatic Control, vol. 52(9), 2007