Exam preparation and "mock exam"

Please refer to the "Exams" document on Nestor for more information on how the exams will be conducted and graded. In place of a full "mock exam," below you will find the reasoning behind the formulation of exams, as well as examples of various exam setups and questions: the former refers to the description of the system(s) under consideration and the latter presents sample questions.

What is the purpose of these exams?

The structure and content of the exams are meant to test whether you have met the learning objectives of the course, with the exception of programming that is the focus of the assignments. Furthermore, the midterm exam is meant to prepare you for the final exam by exposing you to the same type of structure and questions, while also testing your knowledge of the first part of the course.

Quoting from the course description on Ocasys, the learning objectives are the following:

> *At the end of the course, the student is able to:*
> 1. *conceptualize dynamical systems in terms of physical (or virtual) components, the environment in which they operate, as well as their stimuli and responses (outputs)*
> 2. *use mathematics (for example, ordinary and partial differential equations) to describe such dynamical systems*
> 3. *use analytical tools to solve simple cases of dynamical systems across domains*
> 4. *use numerical tools to solve (systems of) ordinary and partial differential equations and be able to discern the accuracy of the solutions*
> 5. *visualize the time histories and phase space solutions of complex dynamical systems*
> 6. *describe in a concise manner the behavior of complex systems supported by evidence*

The exams test whether you have attained learning objectives #1-3 and #5-6. For #4, you may be tested on how to set up numerical solutions (pseudocode) but without actually doing programming; hence, discerning the accuracy of numerical solutions is also not tested in the exams.

How should I study for the exams?

Your source material is the course reader, which compiles all the content covered (to various degrees) during lectures, as well as the preparatory and regular assignments. Preparatory assignments are meant to encourage you to think about the conceptual aspects of the assignments, while the assignments themselves often expand beyond what is covered in the lectures or go into much more depth into specific aspects, while they are obviously rather heavy on the programming aspect. Videos are also made available to you and are meant to help strengthen your understanding of the course material. Ideally, you will have spent sufficient time studying the material in preparation for the assignments before going into the midterm and final exams. Conceptually, you can take any part of the course content and see how the learning objectives relate to it: try to do this mental exercise for the different physical (and virtual) systems we study in this course.

Dynamics of Engineering Systems

One aim for the exam is to challenge you to think away from recipes on how to solve problems; hence, do not expect to receive solved exam questions to help you study. Instead, study to gain conceptual understanding of the (main) characteristics and (most relevant) dynamical behavior of simple engineering systems, develop an intuition of how this behavior can be described –both graphically and in narrative form–, and learn how these systems can be modeled (under specific assumptions).

Examples of exam setups and questions

Disclaimer: The following is not an exhaustive list, i.e. you will probably be asked to conceptualize and analyze different physical (or virtual) systems from the ones in the list, or even combinations of such systems. **The full course content is fair game for the exams** (ref. "How should I study for these exams?")

Note that information will be provided to you without schematics (where possible) in these exams.

- Three particles are constrained to move on a unit circle without colliding. Each particle of mass M_i is driven by torque T_i, and experiences a viscous damping force that opposes its motion and scales with a damping coefficient D_i. Each pair of particles is interconnected with a spring of stiffness k_{ij}.
  a) Draw the free-body diagram of the system defined above and include all necessary information.
  b) Describe qualitatively what you expect will be the behavior of the system as a function of the magnitude of the masses, the damping coefficients and spring stiffness values.
  c) What do you expect will be this behavior if the natural frequencies of the system are numerically close to each other versus when they are strongly dissimilar? Use plots of the kinematics (displacement and velocity time histories) and dynamics (force and other derived quantity time histories) as necessary to support your arguments.
  d) Derive the equation of motion of each particle and present the equation describing the system using indicial notation (i.e. using M_i, D_i, T_i, etc.). List all relevant assumptions used in your model.
  e) Simplify the equation of motion assuming uniformly high viscous damping $D = D_i$ and small masses, and express the equation in terms of the particles' natural frequency ω_i instead of their applied torque T_i.
  f) Write pseudo-code to describe how your model can be solved numerically and briefly discuss expected numerical issues (type of solver, convergence issues, etc.).
  g) How could you model collisions between these particles? Use mathematical expressions to explain your answer.

- A block of mass m rests at its equilibrium position while being connected to a spring of stiffness k that is fixed to a rigid wall installed at the left of the block. The floor on which the block rests moves to the right with velocity v at time $t = 0\ s$.
  a) Assume there is no friction between the block and the floor. Write down the initial conditions of the system and plot the velocities of the block and the floor as functions of time.

b) Let the friction force be nonzero and proportional to the weight of the block by a constant coefficient μ. Sketch the free-body diagram of the system. Plot the kinematics (i.e. the displacement and velocity as a function of time), the phase plot, as well as the time history of the spring force under the assumption that friction is essentially infinite (i.e. $\mu \to \infty$).

c) Using cutting-edge technology, assume we can turn friction on and off at will (i.e. $\mu \to \infty$ for the "on" condition and $\mu = 0$ for the "off" condition). Initially, friction is "on" until we turn it "off" at $t = 0\ s$. If the stiffness of the spring is $16\pi^2\ N/m$ and the mass of the block is 1 kg, how many times does the center of the block cross the equilibrium position during the first 10 seconds? Plot the kinematics and phase plot of the system.

d) Using free-body diagrams, sketch two of the possible states of the system described in part c): positive displacement (block positioned to the right of the equilibrium position) and negative displacement (block positioned to the left of equilibrium). Include and define the internal and external forces acting on the block in each state.

e) Given Newton's second law (the acceleration of an object depends directly upon the net force acting on the object, and inversely upon the mass of the object) derive the equation of motion for the system of parts c) and d).

f) Write pseudocode to show how the behavior of the system described in part e) can be approximated numerically by solving a system of two first-order ODEs (e.g. by calling "ode45"). Make sure to write down explicitly the system of equations to be solved.

- The Bernoulli equation for the steady-state flow of an incompressible fluid (expressed in terms of the total head) has the following form:

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant.}$$

Consider the case of a reservoir containing water ($\rho = 998\ kg/m^3$) draining via a small horizontal pipe of diameter d situated at a distance Δh below the waterline (initial water volume is V).

a) Derive an equation for the volumetric flow-rate (Q) through this pipe as a function of the fluid height. Assume that the flow is steady and friction does not play a role.

b) Assuming that we fit a valve onto the outflow pipe, sketch $Q(t)$ when the valve opening goes from fully closed to fully open over an amount of time Δt, and remains fully open thereafter. Write down the corresponding pseudocode needed to calculate the flow-rate.

c) Sketch $Q(t)$ for a liquid solution with a higher density than water.

Now, consider that we want to collect the outflow of the original reservoir into a second reservoir having an inlet diameter D and whose center (of the inlet), due to design considerations, sits at distance L away from the end of the outflow pipe of the original reservoir.

d) If the instantaneous vertical position of a particle of this outflow jet is given by $y = gt^2/2$, where t is time, derive an expression for the height difference between the outflow pipe center and the inlet of the second reservoir. Assume that the horizontal velocity component is constant.

e) What are acceptable dimensions for the second reservoir's inlet to avoid spills during operation? Hint: how far does the water jet travel as a function of the available hydraulic head?

Finally, consider that the original reservoir filled with solution A flows into the second reservoir that contains solution B. The two species together form the product P with stoichiometry 1:1, and the

reaction is 1st order in both species. The rate of A (mass per second) entering tank 2 depends on the flow-rate and concentration of the solution.

f) Write down the differential equations that describe the evolution of all three compounds in terms of their masses (so, not concentration!)

g) Write pseudocode on how to solve these equations. For the purpose of simplification and convenience, you don't need to capture the change of liquid height in tank 1 and its effect on flow $Q(t)$.

h) Suppose that we can choose between two vessel designs for reservoir 1 that have the same volume V: variant 1, which is wider and shorter, or variant 2 with a smaller diameter and greater height (the cross sectional area of the horizontal outlet pipe would remain the same in both cases). Which would you choose to speed up the process, considering the chemical reaction taking place in reservoir 2? (Assume that the reaction is limited by the supply of species A.)

- Consider a rod of length L and cross-sectional area A that is fixed (horizontally) to a rigid wall on one side; the temperature of the system (rod, wall and their environment) is $60°C$. A rigid mass m impacts the free side of the rod which then deforms by an amount x.

a) Draw the free-body diagram of the mass and write the equation of motion for the mass as it compresses the rod (consider the problem in 1D, i.e. only horizontal movement).

b) Assume that the free end of the rod is subjected to cyclic loading with $x = x_0 e^{i\omega t}$ (i.e. replace the displacement in the equation of section (a) using the new expression given for the displacement). Rewrite the equation of motion in terms of stress and strain and solve for their ratio (σ/ε).

c) Plot the stress-strain response of the rod material for the following cases of cyclic loading: purely elastic material; and, viscoelastic (Maxwell) material.

d) Assuming that the rod material is viscoelastic, set the ratio found in section (b) equal to the complex modulus of a Maxwell material given by $\left[\frac{1}{E} - \frac{i}{\eta\omega}\right]^{-1}$, where E is the elasticity and η is the viscosity, and solve the resulting equation for ω. Using your understanding of the physics of such oscillatory systems, discuss the conditions for which the system exhibits damped vibration or aperiodic damping.

The rigid mass impacting the viscoelastic rod has the geometry of a sphere of radius R that is initially at a temperature of $25°C$. Following impact, the spherical mass sticks to the viscoelastic rod so that heat transfer occurs in the system due to conduction only (in 1D).

e) What are the initial and boundary conditions for the spherical mass?

f) Sketch the 1D temperature distribution along the sphere's center (from $-R$ to R) as a function of time for 5 time increments (the last one for time tending to infinity). For simplicity, assume that the temperature profile in the **rod** remains constant, even after impact.

g) The equation for heat conduction is given by $\frac{\partial T}{\partial t} = \alpha \nabla^2 T$. Write down the equations approximating the behavior of this system in 1D along the centerline of the sphere ($-R \leq x \leq R$) using the second order central difference equation for space and the forward difference equation for time.