Dynamics of Engineering Systems
Final exam

19 - 20

This exam consists of the original exam with the answers behind it. One can first attempt to make the exam and then immediately check the answers.

Disclaimer

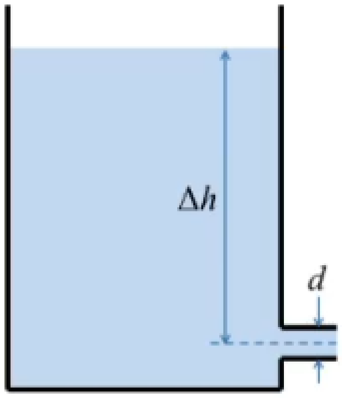
This exam was created using the online explanation the professor gave after the exam. The official answer and grading sheet has not been released at this time, making one unable to officially verify the answers in this sheet. Hence, if one sees incorrect answers within this sheet, please inform Lugus so they can take appropriate actions.

Question 1 (15p)

The Bernoulli equation for the steady-state flow of an incompressible fluid (expressed in terms of the total head) has the following form:

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = constant$$

Let us now consider the case of a reservoir containing water ($\rho = 998\frac{Kg}{m^3}$) draining via a small horizontal pipe of diameter d situated at a distance Δh below the waterline (initial water volume is V)



Derive an equation for the volumetric flow-rate (Q) through this pipe as a function of the fluid height. Assume that the flow is steady and friction does not play a role.

Question 2 (20p)

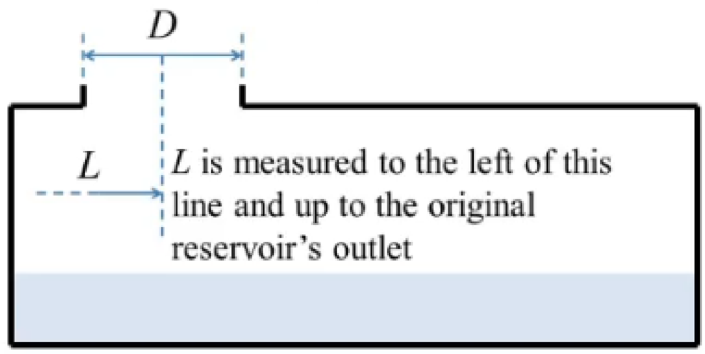
Assuming that we fit a valve onto the outflow pipe, sketch Q(t) when the valve opening goes from fully closed to filly open over an amount of time Δt, and remains fully open thereafter. Write down the corresponding pseudocode needed to calculate the flow-rate.

Question 3 (5p)

Sketch Q(t) for a liquid solution with a higher density than water.

Question 4 (10p)

Now, consider that we want to collect the outflow of the original reservoir in a second reservoir having an inlet of diameter D and whose center, due to design considerations, sits at distance L away from the end of the outflow pipe of the original reservoir.



if the instantaneous vertical position of a particle of this outflow jet is given by $y = \frac{gt^2}{2}$ where t is time, derive an expression for the height difference between the outflow pipe center and the inlet of the second reservoir. Assume that the horizontal velocity component is constant.

Question 5 (15p)

What are acceptable dimensions for the second reservoir's inlet to avoid spills during operation? *Hint: how far does the water jet travel as a function of the available hydraulic head?*

Question 6 (15p)

Finally, consider that the original reservoir filled with solution A flows into the second reservoir that contains solution B. The two species together form the product P with a stoichiometry of 1:1, and the reaction is 1st order in both species. The rate of A (mass per second) entering the second reservoir depends on the flow-rate and concentration of the solution.

Write down the differential equation that describe the evolution of all three compounds in therm of their masses (so not concentration!)

Question 7 (15p)

Wrote pseudocode on how to solve these equations. For the purpose of simplification and convenience you don't need to capture the change of liquid height in tank 1 and its effect on flow Q(t).

Question 8 (5p)

Suppose that we can choose between two vessel designs for reservoir 1 that have the same volume V: variant 1, which is wider and shorter, or variant 2 with a small diameter and greater height (the cross sectional area of the outlet would remain the same in both cases). Which would you choose to speed up the process, considering the chemical reaction taking place in reservoir 2? (Assume that the reaction is limited by the supply of species A)

Question 9 (5p)

Bonus question

We can introduce hydraulic loss Δh_{ls} into the Bernoulli equation to account for the energy loss by the water flowing through the small horizontal pipe. Derive the expression for the hydraulic loss considering two points at the inlet and outlet of the horizontal pipe.

Question 1 (15p)

The Bernoulli equation for the steady-state flow of an incompressible fluid (expressed in terms of the total head) has the following form:

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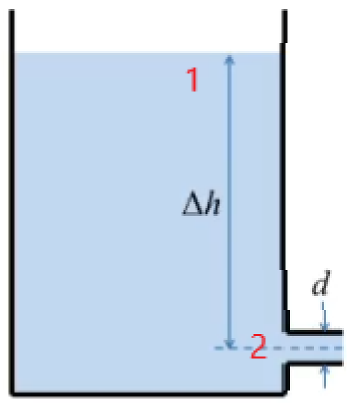
Derive an equation for the volumetric flow-rate (Q) through this pipe as a function of the fluid height. Assume that the flow is steady and friction does not play a role.



The Bernoulli equation allows us to make a comparison in two different situations. One can say that the point on top of the water is point 1, whilst the start of the inlet is point 2. According to the Bernoulli equation, the situations at these points must be constant and thus the same.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

We know that the pressure mentioned is the atmospheric pressure, since the water pressure is transformed into velocity. The difference in atmospheric pressure between points 1 and 2 is insignificant, allowing us to call both P_{atm}. In addition to this, one can assume that the velocity at points 1 is neglectable, since it is over a large area. This gives us:

$$\frac{p_{atm}}{\rho g} + \frac{0^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + \frac{v^2}{2g} + z_2$$

$$z_1 = \frac{v^2}{2g} + z_2 \rightarrow z_1 - z_2 = \frac{v^2}{2g}$$

$$2g(z_1 - z_2) = v^2 \rightarrow v = \sqrt{2g(z_1 - z_2)}$$

$$v = \sqrt{2g(z_1 - z_2)}$$

Flow rate $= Q = A \cdot v$
$A = \pi \cdot r^2 \rightarrow d = \frac{1}{2} \cdot r \rightarrow A = \frac{1}{4}\pi d^2$

$$Q = \frac{1}{4}\pi d^2 \cdot \sqrt{2g(z_1 - z_2)} \quad \vee \quad Q = A \cdot \sqrt{2g(z_1 - z_2)}$$

Question 2 (20p)

Assuming that we fit a valve onto the outflow pipe, sketch Q(t) when the valve opening goes from fully closed to filly open over an amount of time Δt, and remains fully open thereafter. Write down the corresponding pseudocode needed to calculate the flow-rate.

First of all, one can assume that the valve opens with a constant velocity, creating a linear increase of the flow rate until the valve is fully opened. Once the valve is fully open, the water will drain from the reservoir into the outflow pipe, decreasing the overall water level. As one can see in the equation for the flow rate which was determined in question 1, the height of the water in the tank has impact upon the flow rate. Hence, the flow rate will slowly go down in time, until it is almost zero (cause the tank will be empty then).

Question 3 (5p)

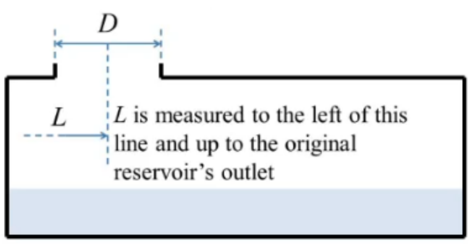
Sketch Q(t) for a liquid solution with a higher density than water.

The equation for the flow rate which was determined in question 1, namely $Q = A\sqrt{2g(z_1 - z_2)}$, does not contain the density of the liquid. As one can see in question 1, the density of the liquid is cancelled out if it is equal at both points. Hence, one can use the drawing made in question 2 to illustrate the flow rate of a liquid with a higher density than water.

Question 4 (10p)

Now, consider that we want to collect the outflow of the original reservoir in a second reservoir having an inlet of diameter D and whose center, due to design considerations, sits at distance L away from the end of the outflow pipe of the original reservoir.

if the instantaneous vertical position of a particle of this outflow jet is given by $y = \frac{gt^2}{2}$ where t is time, derive an expression for the height difference between the outflow pipe center and the inlet of the second reservoir. Assume that the horizontal velocity component is constant.



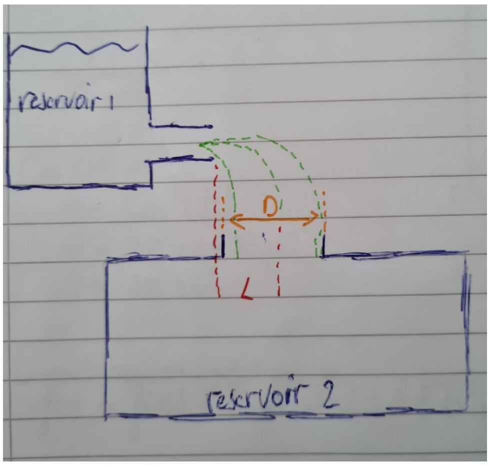
One can assume that the horizontal component is constant, which is the flow rate and the velocity. Since the position x is a function of the velocity at time t, one can say that $x = v \cdot t$
Rewriting this gives us $t = \frac{x}{v}$. One can also say that $x = L$, giving us $t = \frac{L}{v}$.
combining this into one equation gives us $y = \frac{gt^2}{2} \rightarrow \frac{g(\frac{L}{v})^2}{2} \rightarrow \frac{g}{2}\frac{L^2}{v^2}$

$$y = \frac{g}{2}\frac{L^2}{v^2}$$

Question 5 (15p)

What are acceptable dimensions for the second reservoir's inlet to avoid spills during operation?
Hint: how far does the water jet travel as a function of the available hydraulic head?

The question here can be interpreted in many ways, but the original intention was that one has to state the conditions which have to be achieved in order to attain maximal flow rate and minimal spillage.
The first condition that one can derive is can be seen from drawing the situation, as done to the right. One can see that the L line goes from the outlet of the first reservoir towards the exact middle of the inlet of the second reservoir. Since the L goes to the exact middle, one can say that it is exactly half of D which gives us D = 2L. Since we want to prevent spillage, D must **atleast** be 2L, guving us the condition



$$D \geq 2L$$

Secondly, one must take the maximum possible horizontal displacement of the liquid into account. One can see in the picture above the possible flow rates in green, of which one almost spills. To prevent spillage, one can calculate the maximum possible flow rate which can be used to determine the maximum possible horizontal displacement. We start by creating the condition that the diameter of the reservoir must be equal or greater than the possible horizontal displacement:
$D \geq x \rightarrow D \geq t \cdot v \rightarrow D \geq t \cdot \sqrt{2g(z_{1,max} - z_2)}$ As one can see above, z_1 is maximized to get the maximum velocity, since $2, gandz_2$ are constants. This gives us

$$D \geq t \cdot \sqrt{2g(z_{1,max} - z_2)}$$

Bonus Create an expression for L. (No idea why, since it is not stated in the question.)

$$y = \frac{g}{2}(\frac{L}{v})^2 = \frac{g}{2}(\frac{L}{\sqrt{2g(z_1 - z_2)}})^2 = \frac{L^2}{4(z_1 - z_2)}$$

$$y = \frac{L^2}{4(z_1 - z_2)} \rightarrow L^2 = 4y(z_1 - z_2) \rightarrow L = \sqrt{4y(z_1 - z_2)}$$

$$L = \sqrt{4y(z_1 - z_2)}$$

Question 6 (15p)

Finally, consider that the original reservoir filled with solution A flows into the second reservoir that contains solution B. The two species together form the product P with a stoichiometry of 1:1, and the reaction is 1st order in both species. The rate of A (mass per second) entering the second reservoir depends on the flow-rate and concentration of the solution.
Write down the differential equation that describe the evolution of all three compounds in therm of their masses (so not concentration!)

Solution A flows into the second reservoir which contains solution B, so solution B goes down and solution P gets created. In addition, the stoichiometry is 1:1, meaning that the reaction molecules react 1:1. (A + B = P)
$\frac{dMa}{dt}$ = inflow of A - reaction of A with B
$\frac{dMb}{dt}$ = - reaction of A with B
$\frac{dMp}{dt}$ = reaction of A with B

This gives us the following three equations:
$\frac{dMa}{dt} = QC_A - K_1C_AC_BV_2$
$\frac{dMb}{dt} = -K_1C_AC_BV_2$
$\frac{dMp}{dt} = K_1C_AC_BV_2$

Question 7 (15p)

Wrote pseudocode on how to solve these equations. For the purpose of simplification and convenience you don't need to capture the change of liquid height in tank 1 and its effect on flow Q(t).

Note! This question was not covered by the professor, so the answer is not verified. (but it does work in Matlab)

clc; clear; clearvars;
set T_{start}, T_{end} T_{step}, C_A, ,C_B, C_P, K_1, ,V_2 and Q
$y0 = [C_A \;\; C_B \;\; C_P]$;

$Concentrations = @(t, y)ConcSolver(t, y, K_1, Q, V_2)$;
$[t, y] = ode45(Concentrations, [T_{start}; T_{end}], y0)$;
figure(1); plot(t, y);
legend('C_A', 'C_B', 'C_P')

$function[dydt] = ConcSolver(t, y, K_1, Q, V_2)$
$C_A = y(1); \;\; C_B = y(2); \;\; C_P = y(3)$;
$dydt(1) = Q * C_A - K_1 * C_A * C_B * V_2$;
$dydt(2) = -K_1 * C_A * C_B * V_2$;
$dydt(3) = K_1 * C_A * C_B * V_2$;
$dydt = dydt'$;
end

Question 8 (5p)

Suppose that we can choose between two vessel designs for reservoir 1 that have the same volume V: variant 1, which is wider and shorter, or variant 2 with a small diameter and greater height (the cross sectional area of the outlet would remain the same in both cases). Which would you choose to speed up the process, considering the chemical reaction taking place in reservoir 2? (Assume that the reaction is limited by the supply of species A)

As stated above, the only bottleneck for the reaction velocity is the supply of species A. The supply of species A is determined by the flow rate of reservoir 1 into reservoir 2 which has flow rate Q. Question 1 resulted in a flow rate $Q = A\sqrt{2g(z_1 - z_2)}$.
The question states that the pipe diameter A is the same for both vessels and the gravity is also assumed to be the same, which leaves z_1 and z_2. z_2 is the bottom of the reservoir, which is also constant and thus leaves the variable z_1. If one increases z_1, it will increase the force upon the liquid at the pipe and thus the flow rate Q.
Variant 1 is wider and shorter, which decreases z_1. Variant 2, on the other hand, is smaller but taller and thus increasing z_1. Hence, variant 2 of the vessel will increase the flow rate of species A and thus the reaction velocity within reservoir 2.

Question 9 (5p)

Bonus question

We can introduce hydraulic loss Δh_{ls} into the Bernoulli equation to account for the energy loss by the water flowing through the small horizontal pipe. Derive the expression for the hydraulic loss considering two points at the inlet and outlet of the horizontal pipe.

This is a bonus question of which no clear answer was provided. The only hint that was given by the professor is that the answer was given in the first question or part of this document.