Calculus 2 (IEM)

Midterm Exam I

26 February 2021, 14:00–16:00

university of groningen

The exam consists of four problems, worth nine points in total. You get an additional bonus point, so your score will be between 1 and 10. This file contains only Problem 1. Your solution to this problem should be written into a single file, named `problem1.pdf`, with correctly ordered and oriented pages. All files containing your solutions need to be uploaded together, under the `Midterm Exam` assignment in the `Exam - Calculus 2 (IEM)` environment on Nestor, before 16:30 (UTC+1).

Problem 1 (3 points)

Given are three vector functions

$$\boldsymbol{v}_1(t) = \big(t,\, t,\, t^2\big)$$
$$\boldsymbol{v}_2(t) = \big(t^2,\, t,\, t\big)$$
$$\boldsymbol{v}_3(t) = \big(t,\, t^2,\, t\big)$$

where $t \in (-\infty, \infty)$.

a) Determine the volume $V(t)$ of the parallelepiped spanned by the vectors $\boldsymbol{v}_1(t)$, $\boldsymbol{v}_2(t)$, $\boldsymbol{v}_3(t)$.

b) Verify that $V(-1) = 4$ to make sure your result in a) is correct.

c) Determine the values of t for which $V(t)$ is zero.

d) Determine the values of t for which $V(t)$ achieves a local maximum. [You must give a complete argument for full points.]

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Problem 2 (2.5 points)

The position of a particle at any moment $t \in [0, \infty)$ is given by

$$\mathbf{r}(t) = \big((t-1)\sin t + (t+1)\cos t\big)\mathbf{i} + \big((t+1)\sin t - (t-1)\cos t\big)\mathbf{j}.$$

Determine the following as a function of t:

a) the length $s(t)$ of the path traversed by the particle;

b) the curvature $\kappa(t)$ of the path;

c) the tangential and normal components of acceleration.

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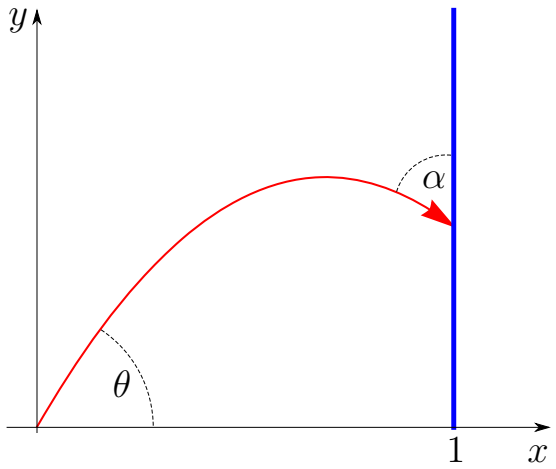
Problem 3 (2.5 points)

A projectile is fired from $(0, 0)$ at an angle $\theta = \frac{\pi}{3}$ and speed $v_0 = 4$. At the point $(1, 0)$ a vertical wall $W = \{(1, t) \ : \ t \geq 0\}$ is positioned. Assume that (gravitational) acceleration is given by the constant vector $\boldsymbol{g} = -10\mathbf{j}$ at every point.

a) Determine the position $\mathbf{r}(t)$ of the projectile at time t.

b) Determine the time and point of impact with the wall.

c) Determine the value of the angle α at which the projectile hits the wall. [You may use your calculator to evaluate trigonometric functions.]



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The exam consists of four problems, worth nine points in total. You get an additional bonus point, so your score will be between 1 and 10. This file contains only Problem 4. Your solution to this problem should be written into a single file, named `problem4.pdf`, with correctly ordered and oriented pages. All files containing your solutions need to be uploaded together, under the `Midterm Exam` assignment in the `Exam - Calculus 2 (IEM)` environment on Nestor, before 16:30 (UTC+1).

Problem 4 (1 point)

Describe (classify) and sketch the following quadric surface:

$$x^2 + 2y^2 - z^2 - 2x + 4y - 4z + 3 = 0.$$

[**Hint:** Consider completing squares and changing variables by translation.]