

UNIVERSITY OF GRONINGEN

COURSE SUPPORT READER

Fluid Dynamics

Author:

Tim Kousemaker (2016)

Paul van den Tempel (2019)

The logo for Lugus, featuring the word "Lugus" in a bold, green, sans-serif font. The letter 'L' is significantly larger than the other letters, and the 'u' has a distinctive shape with a small loop.

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This information may give the impression to be a complete and balanced reflection of the subjects being treated in the course. However, it can only be regarded as an additional aid in preparation for the exam. It is in no way a substitute to the course literature or the lectures and tutorials offered in the course.

Emphasizing the importance of active participation by students in the lectures, we decided to provide this information shortly before the final exam of the course. After the resit the package will be taken off line. Each year, the packages are revised and adapted to the course schedule and literature for the course of that particular year. We strongly discourage to use packages of the previous years.

Contents

1	Introduction	3
2	Geometrical Expressions	4
3	Balances	4
3.1	Mass Balance	5
3.1.1	Mole Balance and Chemical Reactions	6
3.2	Energy Balance	7
3.2.1	Heat balance	8
3.2.2	Mechanical energy balance	8
3.3	Momentum Balance	8
4	Dimensional Analysis	9
4.1	Dimensionless Numbers	11
4.1.1	Reynolds Number	11
5	Drag Force	12
6	Fluid flow	13
6.1	Extended Bernoulli and the Bernoulli equation	13
6.2	Friction	13
7	Heat transport	14
7.1	Conduction	15
7.1.1	Conduction through flat plates and the Overall heat transfer coefficient . . .	15
7.1.2	radial coordinates	16
7.2	Convection	16
8	Mass Transport	17
8.1	Similarities With Heat Transport	17
9	Diffusion flow in mass balances	18
10	Practice Questions	19

1 Introduction

This document is an overview of the contents of the course Fluid dynamics, it is being offered to you by TBV Lugus. This document contains information from the study book as well as the lectures. This reader can be used as study material for the course, it is however always important to also study the sheets and the book as well. I strongly recommend to practice as much as you can, for example by making old exams and making all tutorials. First, try to make them on your own without looking at the solutions. When solving a problem for the second time, think about why you chose this approach for the solution. The hardest part of this course is choosing where to start! In general it is important to ask yourself the following questions:

1. What is the problem?
2. Which variables must be solved and which variables are constant?
3. Which formulas describes the situation? You should always start with drafting a balance.
4. See which units you can already fill in.
5. Which units are missing? Can these be calculated?
6. Can you solve the equations?

Some remarks:

- This document is not officially part of the course.
- This document is meant to help you, but might be missing parts.
- This document is not a replacement for studying the book.
- Practice is the most important way to familiarise yourself with the course content.

The math involved in physical transport phenomena is in general very elementary. Basically, if you know the rules shown below you should be able to solve almost all (differential) equations:

$$\int a dx = ax + c$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

$$\int \frac{dx}{x} = \ln(x) + c$$

$$\lim_{\delta \rightarrow 0} \frac{f(x) - f(x + \delta)}{\delta} = \frac{\partial f(x)}{\partial x}$$

$$\frac{df}{dx} = f \quad \Rightarrow \quad f = \exp(x) + c$$

Important notice

This text is far from a dictation to the course "Physical Systems for I.E.M.", let alone being an official document. Therefore note the following remarks:

- This document does **not contain the official lecture notes** regarding the course "Physical Systems for I.E.M."
- This document does **not reflect all subjects being treated** in the course
- This document does **not substitute the book(s)** used in the course
- This document is only **meant as an aid to solve certain exercises**; for other exercises see the book or reader in use!

It is very important to practice integration, as easily forgotten rules such as the chain rule also make their return in this course. It is strongly recommended to revise these rules.

2 Geometrical Expressions

Let us start with a list of all the geometrical expressions for areas and volumes of circles, spheres and cylinders in terms of diameter or radius. It is important to be able to use these equations comfortably and to remember them by head. In almost every exercise it is required to use at least one of these expressions! (A = Area, S = Circumference, V = Volume)

Circles:

$$A = \frac{1}{4}\pi D^2 = \pi R^2 \quad S = D\pi = 2\pi R \quad (1)$$

Spheres:

$$V = \frac{1}{6}\pi D^3 = \frac{4}{3}\pi R^3 \quad A = \pi D^2 = 4\pi R^2 \quad (2)$$

Cylinders: Bottom and top of cylinder not included for area. Usually only the side is relevant (example: heat loss in pipelines to the environment)

$$V = \frac{1}{4}\pi D^2 L = \pi R^2 L \quad A = D\pi L = 2\pi RL \quad (3)$$

3 Balances

A lot of exercises start with a balance. The first tutorials are meant to help you practice with this. A general way to tackle problems like this is to do the following:

1. Make a drawing of the situation, including all inflows and outflows.
2. Select the parameter of interest (X) which is to be transported.
3. Define the Control Volume (CV).
4. Determine whether or not X changes over time (is it steady state?)

The terms 'stationary' or 'steady state' imply that the variable of interest is not changing over time in the system. For instance, the system is in steady state when there is no accumulation inside (i.e. quantity inside the control volume is not dependent on time).

The general definition of a balance is:

$$\frac{dVX}{dt} = in - out + production \quad (4)$$

This can be written as:

$$\frac{dVX}{dt} = \phi_v X|_{in} - \phi_v X|_{out} + r_x V \quad (5)$$

in which ϕ_v is the volumetric flow [m³/s], r_x is the reaction rate (or in general, production rate) per cubic meter and V is the volume [m³]. X is a quantity that is dependent on the kind of balance being made (see table 1).

Balance type	X	Units
mass balance	ρ	kg/m ³
energy balance	ρe	J/m ³
heat balance	$\rho C_p T$	J/m ³
momentum balance	ρv	N.s/m ³

Table 1: Specifications of X in the general balance equation, yielding the corresponding balance.

3.1 Mass Balance

Substituting $X = \rho$ in equation 5, we obtain the mass balance from the general expression of a balance:

$$\frac{dVX}{dt} = \phi_v \rho|_{in} - \phi_v \rho|_{out} + r_x V \quad (6)$$

This equation can be written in different forms; Note that $V\rho = M$ and $\phi_v \rho = \phi_m$.

If there is no production (chemical reaction) of a component in the system, the production term cancels out and thus we obtain:

$$\frac{dV\rho}{dt} = \phi_v \rho|_{in} - \phi_v \rho|_{out} \quad (7)$$

Constants in the numerator of the accumulation term (on the left) can always be put in front. Sometimes a valid assumption to make is that the density of the ingoing and outgoing stream are all equal and constant. This can be the case for a large water stream with a low concentration of component X. Be always aware of the assumptions you make!

If the density is constant, the following is true as well:

$$\rho \frac{dV}{dt} = \phi_v \rho|_{in} - \phi_v \rho|_{out} \quad (8)$$

In this case we see that the mass balance can be developed to a volume balance. This may be useful to quickly obtain insights in unknown sizes of streams.

$$\frac{dV}{dt} = \phi v_{in} - \phi v_{out} \quad (9)$$

3.1.1 Mole Balance and Chemical Reactions

If values in the problem are known in terms of concentration (mol/m^3) and/or a chemical reaction takes place, it is useful to use a mole balance. The mole balance can be derived from the mass balance in the following way:

$$\frac{dM}{dt} = \phi m|_{in} - \phi m|_{out} + R_a V M_a \quad (kg/s) \quad (10)$$

This is a mass balance of component A including a chemical reaction with M_a the molecular weight of A. By dividing all terms by the molecular weight, the mole balance is obtained (F is used for mole flow):

$$\frac{dF_a}{dt} = F_a|_{in} - F_a|_{out} + R_a V (mol/s) \quad (11)$$

Subsequently this can be written in terms of concentration:

$$\frac{dC_a V}{dt} = \phi v C_a|_{in} - \phi v C_a|_{out} + R_a V \quad (12)$$

with C_a the concentration of component A in mol/m^3 . Make sure that all the terms in a balance are in the same units! When working with a chemical reaction, the term $R_a V$ starts to play a role. The expression of R_a depends on the reaction. For a reaction where A reacts with B to form P, it looks like the following:

$$R_a = -k C_a^\alpha C_b^\beta \quad (13)$$

In here, α and β are the rate orders of component A and component B, respectively and k is the rate constant. The rate order of a component can also be zero and is usually referred to as 'the reaction is zero order'. Think about the minus sign of R_a . Is component A being formed or does it 'disappear' in a reaction?

In this course it is important to know that there are three reactor types: Batch, CISTR and PFR. All are schematically shown in figure 1.

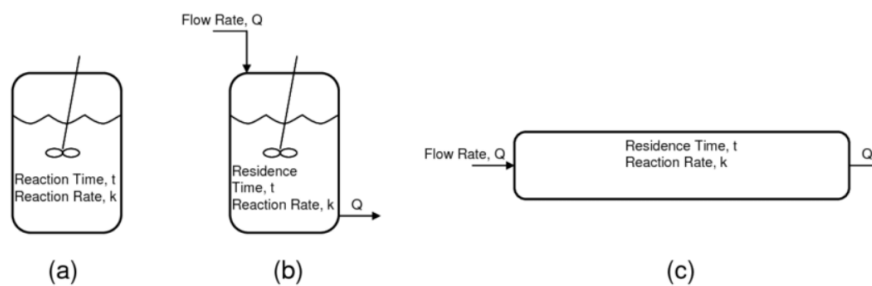


Figure 1: *Three types of reactors: Batch (a), CISTR (b) and PFR (c)*

The slides give an overview of the equations that correspond to every kind of reactor. Some important remarks:

- A batch does not contain an inlet and outlet. There is always accumulation inside a batch reactor once a reaction takes place inside it (component removed or formed by the reaction).
- A semi-batch is a mix between a CISTR and a normal batch, as a semi-batch contains either only an inlet or outlet, but not both.

- In this course, a CISTR is always considered to be stationary (steady state). This means: The concentration of a component is constant over time in the reactor.
- As batch and CISTR reactors are ideally stirred, the concentration in the tank is the same at every position inside the tank. Therefore it may be assumed that the concentration in the outlet of a CISTR is also the same as the concentration inside the CISTR.
- A PFR is not ideally stirred! Therefore a concentration profile exists over the length of the tube. It is important to understand how to find profiles inside the PFR. For a work-out of this, revise the lecture about chemical reactions.

Sometimes the performance or the extent of a reaction is expressed in terms of conversion. The conversion is equal to:

$$X_a = \frac{C_{a,0} - C_a}{C_{a,0}} \quad (14)$$

with X the conversion of component A, and $C_{a,0}$ the inlet concentration and C_a the concentration at the outlet of the reactor (in case of PFR and CISTR) or at time t (for a batch).

It is very important that you can apply the mass balance easily. Practice until you feel comfortable with all forms of the mass balance.

3.2 Energy Balance

From the general balance the energy balance can be written as:

$$\frac{dV\rho e}{dt} = \phi_v \rho e|_{in} - \phi_v \rho e|_{out} + \phi q + \phi w + P \quad (W) \quad (15)$$

in this equation ϕq is the heat added to (or removed from) the system (for example by a heating element), ϕw is the work performed on the control volume, for example by a piston, . P is the production of heat for example by a chemical reaction or due to friction. There are different 'energy concentrations (e)' (all in units J/kg). These are expressed in table 2.

Table 2:	
Energy form	energy concentration
kinetic energy	$1/2v^2$
potential energy	gz
internal energy	u
pressure energy	p/ρ
enthalpy	$h (= u + p/\rho)$

In this way, the total energy concentration can be written as:

$$e = u + \frac{p}{\rho} + \frac{1}{2}v^2 + gh \quad (16)$$

This will result in the overall energy balance on the formula sheet.

3.2.1 Heat balance

In this course it is important to make good decisions in how you will start to solve a problem. The energy balance discussed in the previous section is a combination of all energy terms, but not all of them are usually relevant for a problem.

For problems involving temperature differences and accumulation, the heat balance may be a useful tool to employ. This balance only takes into account thermal energy. By filling in $X = \rho C_p T$ in the general balance (equation 5), we obtain the following:

$$\frac{dV \rho C_p T}{dt} = \phi v \rho C_p T|_{in} - \phi v \rho C_p T|_{out} + \phi q + P \quad (W) \quad (17)$$

Note that every term in this equation has the units Joules/s (or Watt). Usually the outlet temperature (T_{out}) is assumed to have the same temperature as the CV itself. This is only true if the control volume is ideally stirred, meaning that the same conditions (temperature and concentration) apply everywhere in the system. The inlet does not necessarily have the same conditions as the CV!

P typically consists of the heat production inside the tank, but might as well contain terms for additional heating elements, if present. Note that this replaces the $r_x V$ term in equation 5.

3.2.2 Mechanical energy balance

The mechanical energy balance can be seen as a total energy balance, but without thermal energy in it. By choosing two points (similarly as in Bernoulli related problems), this can be used to determine mechanical parameters, such as pressure drop, pump power and energy dissipation due to friction.

$$\frac{dE}{dt} = \phi_m \left[\frac{p_1 - p_2}{\rho} + g(h_1 - h_2) + \frac{1}{2}(v_1^2 - v_2^2) \right] + \phi w - e_{fr} \phi_m \quad (18)$$

In the mechanical energy balance, ϕw can for example be work performed by a pump. More on this will be discussed later in section 6.1.

3.3 Momentum Balance

Momentum is essentially a quantity of movement of an object. Momentum can be added or removed from a system by means of inlet flow, outlet flow or additional forces acting on or by the system. It is a conserved quantity and cannot be generated by definition.

The momentum balance can be obtained by filling in $X = \rho v$ in the general balance (equation 5), with v the velocity.

$$\frac{dMv}{dt} = \phi m v|_{in} + \phi m v|_{out} + \Sigma F \quad (N) \quad (19)$$

Knowing $\phi m = \phi v \rho = v A \rho$, this can be written as:

$$\frac{dMv}{dt} = \rho A v^2|_{in} + \rho A v^2|_{out} + \Sigma F \quad (N) \quad (20)$$

When working with a momentum balance, it is very important to be aware of the following:

1. Momentum is a vectorial quantity. One momentum balance can be set up for only one direction. Therefore, it only includes forces and momentum acting in the same axis.

2. Note that the accumulation term on the left is nothing more than a resulting force. (the term reads $m * dv/dt = m * a$). If the system is at constant velocity, there is no acceleration and thus the accumulation term is zero (steady state).
3. The sign of every term in the momentum balance has to be considered carefully by looking at the direction of the force or momentum.

An illustration is given to clarify the last comment. Let's consider the following system:

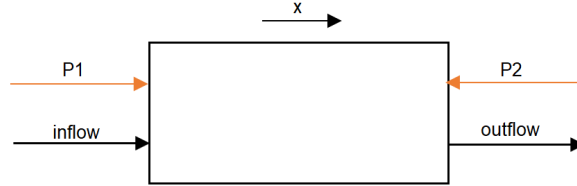


Figure 2: A theoretical CV with indicated forces and flows.

A force or flow can be both in the positive or negative (indicated) direction. Furthermore, a force can be exerted on the system (added momentum or force), or be exerted by the system (removed momentum or force). Instinctively it makes sense that forces acted on the system would gain a plus sign and the ones removed from the system a minus sign. However, the signs switch once a force is in the opposite direction from the one indicated in the figure. In summary, table 3 is constructed. It is highly recommended to practice this and check the result of the given figure above.

Table 3:

	On CV	By CV
pos. direction	+	-
neg. direction	-	+

The momentum balance for the system shown in figure 2 is:

$$\frac{dMv}{dt} = \phi mv|_{in} - \phi mv|_{out} + P_1 A - P_2 A \quad (21)$$

Note that every term in the momentum balance has the units of force (Newton). That is why the pressure has to be multiplied with the area on which the pressure is exerted to obtain the units Newton. In general, forces acting on the system always include pressure forces at both boundaries, but also normal forces and gravitational force.

4 Dimensional Analysis

The intensively used dimensionless numbers in this course indicate the relative importance of two transport mechanisms. They were all derived by doing an dimensional analysis. The general way to perform a dimensional analysis is as follows:

1. Choose a quantity which needs to be researched (X)
2. Determine the variables that influence the quantity chosen by 1 ($X = f(a,b,c,..)$)

3. Express all parameters (X,a,b,c..) in basic dimensions (m,s,kg,K,cd)
4. Write in the form of $X = K (a^\alpha, b^\beta, c^\gamma \dots)$
5. Solve the exponent matrix and rearrange to obtain your dimensionless number(s)
6. optional: Check result with Buckingham π Theorem: Number of dimensionless groups = Number of dimensional quantities – Number of basic dimensions.

Steps one and two are usually already done. It is up to you to express all parameters in their units and find the dimensionless numbers in this way.

As an example, let's consider the upscaling of a mixer. It is determined that the stirring power P (W) is a function of the rotational speed of the stirrer N (s^{-1}), the density of the medium ρ ($kg\ m^{-3}$), the viscosity of the medium η ($Pa\ s$) and the diameter of the reactor D (m) (Practice Question 1). First, this sentence can be written as:

$$P = f(N, \rho, \eta, D) \quad (22)$$

Watts, Joules and Pascal can be broken down to the three basic units meters, seconds and kg. A helpful support is to consider the following:

- $1\ W = 1\ J\ s^{-1}$
- $1\ J = 1\ N.m\ ()$
- $1\ Pa = 1\ N.m^{-2}$ (Pressure equals F/A)
- $1\ N = 1\ kg.m.s^{-2}$ (think about the formula $F = m \cdot a$)

Substituting the parameters for their corresponding units, this will give:

$$kg\ m^2\ s^{-2} = (s^{-1})^a (kg\ m^{-3})^b (kg\ m^{-1} s^{-1})^c (m)^d \quad (23)$$

By setting all units equal, we obtain the following exponent matrix:

$$kg : 1 = b + c \quad m : 2 = -3b - c + d \quad s : -2 = -c$$

As can be seen, this includes 3 equations with 4 unknowns, so this cannot be solved directly! The way to proceed here is to express 3 of the four unknowns in one parameter. The easiest way to continue is to express everything in terms of c (appears in all three equations). This results in:

$$a = 3 - c \quad b = 1 - c \quad c = c \quad d = 5 - 2c$$

And so:

$$P = K\ N^{3-c}\ \rho^{1-c}\ \eta^c\ D^{5-2c} \quad (24)$$

Rearranging this gives:

$$\frac{P}{N^3 \rho D^5} = K \left(\frac{\eta}{N \rho D^2} \right)^c \quad (25)$$

The two dimensionless numbers are now visible! The reason why dimensional analysis is so useful is because the obtained dimensionless groups are constant for every situation. This is often used for

upscaling. In this example, the value of the dimensionless numbers for a small sized mixer would be the same as this number for a big sized mixer:

$$\left(\frac{P}{N^3 \rho D^5}\right)_{small} = \left(\frac{P}{N^3 \rho D^5}\right)_{big} \quad (26)$$

Plenty of examples are given in the slides of this course. What is important to note, is that the proportionality constant (K) is not a dimensionless number on its own, but rather tells something about the relation between them (in this example it is a ratio between the dimensionless numbers).

4.1 Dimensionless Numbers

The most important dimensionless numbers in this course are listed below. Revise the course lectures for more information on the physical meaning of all these numbers. Note that in the formula for the Prandtl and Schmidt number, the ν is the *kinematic viscosity*, not the velocity! ($\nu = \eta/\rho$)

$$Re = \frac{\rho v D}{\eta} \quad Nu = \frac{h D}{\lambda} \quad Pr = \frac{\nu}{a} \quad Sc = \frac{\nu}{D_a} \quad Sh = \frac{k D}{D_a} \quad (27)$$

4.1.1 Reynolds Number

For the following sections, the Reynolds number plays an important role in the calculations. In this section, some light is shed on the functionality of this number.

The Reynolds number gives an indication of the behavior of your flow. For very low Reynolds numbers, the fluid shows *laminar* behavior. For high Reynolds numbers (turbulent flow), the fluid starts to form eddies (small vortexes) (figure 3).

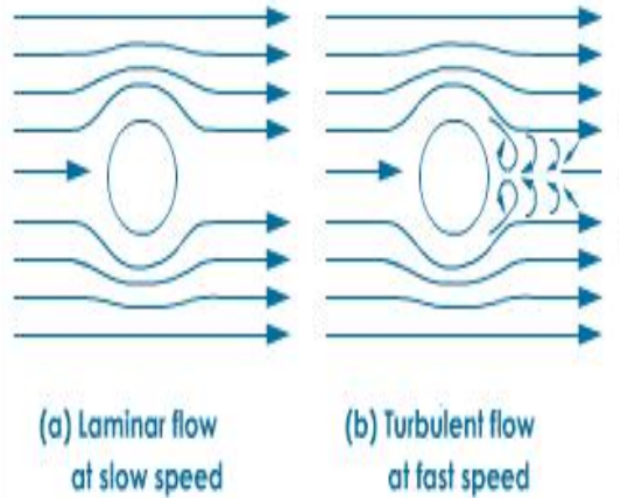


Figure 3: An illustration of laminar and turbulent flow

In the expression of reynolds (equation 27), v is the velocity ρ is the density of the liquid or gas that is flowing, η is the dynamic viscosity and D the diameter of *the flow*. In case of a flow through a tube, naturally the diameter of the tube can be taken. For a very large tank with a very small particle sinking in it, it makes more sense to use the diameter of the particle instead the diameter of

the tank. After all, the fluid is only flowing very near to the particle! Concluding, think carefully what the diameter of your actual flow is, and use that in the Reynolds number.

5 Drag Force

Drag force is the force that is experienced if cycling in headwind, when skydiving, when standing below a waterfall etcetera. In all these cases, the drag force is caused by a relative velocity difference between the object and the medium surrounding it. In this course, a typical example that is used is a sinking or falling sphere. We always consider at least three forces in the vertical direction for falling or rising objects.

- Gravitational force
- Buoyancy force
- Drag force

The gravitational force equals:

$$F_g = M_p g = \rho_p V g \quad (28)$$

In this expression, ρ_p is the particle density and M_p the particle mass. The buoyancy force is caused by the tendency of liquid to occupy the space taken by the object in the medium. This is also known as the force that causes objects to float in water (floating force would also be a suitable name). The formula is very similar to the gravitational force, but the density of the medium is used (ρ_l):

$$F_b = M_l g = \rho_l V g \quad (29)$$

The drag force is dependent on the area of the object perpendicular to the flow ($A_\perp$). The direction is dependent on the flow against the object, which might not necessarily be in the same direction as the object moves. It might also retard the movement of an object! A practical example is the wind pushing back if you cycle in headwind. In case of sinking spheres, the drag force is also directed upwards. In an extreme case: the drag force that is directed upwards causes you not to accelerate any further when skydiving. The expression for the drag force is:

$$F_D = C_D A_\perp \frac{1}{2} \rho_l v^2 \quad (30)$$

Note that here, C_D is the drag force coefficient, the density used in this expression is the one for the medium, not of the particle. $A_\perp$ for a sphere is the maximum perpendicular area on the flow, which is $\frac{1}{4}\pi D^2$ (area of a circle).

The drag force coefficient is dependent on Reynolds. We distinguish two regions in this course:

$$C_D = 24/Re \quad Re < 0.1 \quad C_D = 0.43 \quad (10^3 < Re < 10^5) \quad (31)$$

In general: the following procedure can be used to solve questions including drag force:

1. Make a Force balance over your object in the relevant direction (including F_g , F_b and F_D , but maybe also additional forces, depending on the situation).
2. develop your force balance by filling in the definitions of your forces.

3. In case the velocity is unknown and the goal is to determine this, assume laminar or turbulent flow to be able to calculate the drag force coefficient. (do you expect high or low velocities?)
4. Substitute the equation of your chosen C_d equation and substitute into the expression for F_d .
5. After determining the velocity, check your assumption by calculating Reynolds. Always check your assumption!

6 Fluid flow

6.1 Extended Bernoulli and the Bernoulli equation

For very long pipelines and channels, friction starts to play a significant part in the calculation. For this the extended Bernoulli equation (equation 18) can be used (see section 3.2.2).

For situations where friction can be safely ignored, the normal bernoulli equation can be applied to obtain information on pressure drops, velocities or height differences:

$$\frac{p}{\rho} + g(h) + \frac{1}{2}(v^2) = Constant \quad (32)$$

or when taking two connected points in a system:

$$\frac{p_1 - p_2}{\rho} + g(h_1 - h_2) + \frac{1}{2}(v_1^2 - v_2^2) = 0 \quad (33)$$

When using Bernoulli, always indicate which two points are used in the drawing or figure (if there is no drawing shown, make a drawing yourself!). Usually, the hardest part of bernoulli related problems is to decide which two points should be used. Some convenient options are considered:

- By choosing two points that are directly in contact with the atmosphere (1.013 bar, p_0) the pressure term in equation 33 cancels out ($p_1 = p_2 = p_0$).
- By choosing a point inside a vessel or container with a large cross-sectional area, it may be assumed that the velocity in this point is approximately zero (provided that this cross-sectional area is much bigger than the area of the inlet or outlet stream).
- By choosing two points inside a pipeline, it is known that the velocities in both points are equal, provided that the cross-sectional area of the pipeline is constant and that it is a stationary flow (otherwise accumulation in pipeline).

6.2 Friction

In a long pipeline or channel, energy is dissipated due to friction. In fact, this energy 'loss' is converted into heat. To start making calculations on friction loss, we first consider the mechanical energy balance:

$$\frac{dE}{dt} = -\phi_m \left[\frac{p_2 - p_1}{\rho} + g(h_2 - h_1) + \frac{1}{2}(v_2^2 - v_1^2) \right] + \phi w - e_{fr} \phi_m \quad (34)$$

It is good to use the habit of choosing point 1 somewhere at the startpoint of your system, and point 2 somewhere further downstream. In this way, the minus sign in front of the friction term is valid (energy lost). The term ' e_{fr} ' (in J/kg) can be calculated according to the following formula:

$$e_{fr} = \sum_i \left(f \frac{1}{2} v^2 \frac{SL}{A} \right) + \sum_j K_w \frac{1}{2} v^2 \quad (35)$$

In this equation, f is the friction factor, ρ the density of the fluid, v the velocity, S the 'wetted perimeter' and A the wetted surface area of the stream. For illustration, figure 4 is included. The red line indicates the wetted perimeter.

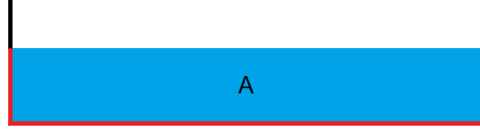


Figure 4: Illustration of the etted perimeter S (in red) and the wetted surface area A (blue).

The second summation takes into account additional friction caused by bends, valves etc., in which K_w is a constant that is dependent on the kind of irregularity.

For straight tubes or channels, the friction loss in J/kg is usually written as the following:

$$e_{fr} = 4f \frac{1}{2} v^2 \frac{L}{D_H} \quad (36)$$

with D_H the hydraulic diameter. This allows us to make calculations for every channel shape. It is defined as:

$$D_H = \frac{4A}{S} \quad (37)$$

substituting this in equation 36 gives back the first term of equation 35. It turns out that the hydraulic diameter for a circular pipeline is actually equal to the diameter of the pipeline itself ($D_H = D$). Try to figure out why!

The friction factor is in fact dependent on the Reynolds number and therefore the velocity. But when the velocity is unknown, there is no way to proceed from here unless an assumption is made. We consider two regions:

$$4f = 64/Re \quad (Laminar\ flow) \quad (38)$$

$$4f = 0.316 Re^{-1/4} \quad (Turbulent\ flow) \quad (39)$$

When very low velocities are expected, the laminar region can be assumed first. Of course it is required to check your assumption by calculating Reynolds! If it turns out that the wrong assumption has been made, then the other region has to be chosen and the calculation has to be performed again.

7 Heat transport

In general, two different forms of heat transport are considered:

- *conduction*: Heat flow or flux (flow per m²) through walls or stagnant media.
- *convection*: Heat flow or flux due to an interaction with a flowing medium, that takes away heat.

7.1 Conduction

The key formula for conduction through walls or stagnant media is the Fourier equation:

$$\phi q'' = -\lambda \frac{dT}{dr} \quad \phi q = -\lambda A \frac{dT}{dr} \quad (40)$$

As shown, the fourier equation can be either expressed in flux (left) or in flow (right). λ represents the *thermal conductivity* of a material (in W/(m K)). Materials that show less resistance for heat (read, heat and cool down easily) have a very high thermal conductivity, as this increases the heat flow. Isolation materials have a very low thermal conductivity and show therefore more resistance for heat flow. The formula can be interpreted as the following: a heat flow (ϕq) or flux ($\phi q''$) is caused by a certain temperature difference or gradient at that specific point (in radial coordinates: dT/dr , for flat surfaces dT/dx) and the ease of heat penetration through a material is dependent on the kind of material itself. The easier the material can let through heat, the higher the thermal conductivity λ

7.1.1 Conduction through flat plates and the Overall heat transfer coefficient

First, let us take the heat flow through a wall of an oven, that consists of two layers (figure 5).

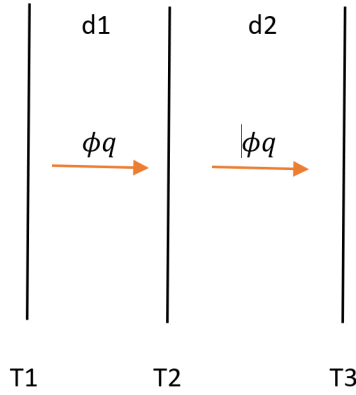


Figure 5: Heat flow through a two layered wall.

In that case, the heat flux can be written in terms of absolute differences (T_1 as the highest temperature and T_3 as the lowest).

$$\phi q'' = \lambda \frac{\Delta T}{\Delta x} \quad (41)$$

and with d the thickness:

$$\phi q_1'' = \lambda_1 \frac{T_1 - T_2}{d_1} \quad T_1 - T_2 = \phi q_1'' \frac{d_1}{\lambda_1} \quad (42)$$

similarly for the second layer, we obtain

$$\phi q_2'' = \lambda_2 \frac{T_2 - T_3}{d_2} \quad T_2 - T_3 = \phi q_2'' \frac{d_2}{\lambda_2} \quad (43)$$

In steady state conditions, the heat flux through the first layer equals the heat flux through the second layer! If this is not steady state, the wall would cool down or heat up (temperature dependent on time, non-steady state):

$$\phi q_1'' = \phi q_2'' = \phi q'' \quad (44)$$

When rewriting the two equations for the temperature differences (as shown above) and adding them up, we obtain the expression for the final heat flux:

$$\phi''q = \frac{T_1 - T_3}{\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2}} \quad (45)$$

The terms in the denominator can be regarded as resistances. We now introduce the overall heat transfer coefficient U ($\text{W}/\text{m}^2 \text{ K}$), which is a number that takes into account all these resistances:

$$U = \frac{1}{\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2}} \quad (46)$$

In this way, the formula for heat flux (equation 46) through a wall can be written as:

$$\phi q'' = U \Delta T \quad \text{or} \quad \phi q = U A \Delta T \quad (47)$$

If the overall heat transfer coefficient is given in the question, this equation can be directly implemented in the heat balance and used to calculate the heat loss to the environment (see equation 17).

7.1.2 radial coordinates

For radial coordinates (spherical or cylindrical), the Fourier equation has to be applied in a slightly different way, as the area through which the heat is flowing is now position dependent. As an example, we take a spherical object. The area through which heat flows (from inside to outside) is πD^2 or $4\pi r^2$ (see section 2).

In that case, equation 40 develops to:

$$\phi q = -\lambda 4\pi r^2 \frac{dT}{dr} \quad (48)$$

In this way the fourier equation can be rearranged and integrated over the radius. Using two boundary conditions $r = R_1 \rightarrow T = T_1$ and $r = R_2 \rightarrow T = T_2$, we obtain:

$$T_1 - T_2 = \frac{\phi q}{4\pi\lambda} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (49)$$

By setting this equation up for multiple layers again, the final formula for the heat flux can be similarly obtained as in section 7.1.1.

7.2 Convection

Convection is additional heat transport by interaction with a flowing medium. This leads to additional terms in the denominator (or resistances) in equation 46. Again, an illustration will be given. The word 'convection' is generally used for heat transport caused by a flow.

Suppose we have a flow on each side of a two-layered wall (the same wall as in section 7.1.1). In this case we need to include additional equations that describe the heat flow from the wall to the bulk temperature of the flowing medium outside the wall. we use Newton's equation for interaction of flow with a wall (equation 50). This can both be set up for flow on the inside of the wall and flow on the outside of the wall:

$$\phi q = h A \Delta T \quad \Delta T = \frac{\phi q}{h A} \quad (50)$$

where h is the heat transfer coefficient (W/m^2K). As shown here, this equation can also be rewritten for the temperature difference. This can then be incorporated into the final heat flow expression by adding this to equations 42 and 43 to obtain:

$$\phi''q = \frac{\Delta T}{\frac{1}{h_{inside}} + \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{1}{h_{outside}}} \quad (51)$$

Again, if an overall heat transfer coefficient (U) is given, all these resistances are already calculated for you and combined into one constant. Also this equation can then be rewritten to equation 47. A heat transfer coefficient for convective heat transport is correlated to the Nusselt number, and dependent on the diameter of the sphere:

$$Nu = \frac{hD}{\lambda} \quad h = \frac{Nu \lambda}{D} \quad (52)$$

The Nusselt number is in turn determined from Reynolds and the Prandtl number:

$$Nu = 0.027Re^{0.8}Pr^{0.33} \quad Re > 10^4, \quad Pr > 0.7 \quad (53)$$

and for flow around spheres:

$$Nu = 2.0 + 0.66Re^{0.5}Pr^{0.33} \quad 10 < Re < 10^4, \quad Pr > 0.7 \quad (54)$$

For a sphere with no flow around it, the Nusselt number is 2! This may also be assumed once Reynolds is lower than 10. In this way, the heat transfer coefficient can still be determined even though it is not given.

8 Mass Transport

8.1 Similarities With Heat Transport

Mass transport shows many similarities with heat transport. we compare the equations that describe the driving forces of heat and mass transport:

$$\phi m'' = k\Delta c \quad \phi q'' = h\Delta T \quad (55)$$

in this equation, k is the mass transfer coefficient (m/s) and Δc the concentration difference. This is an equation that describes the mass transport (in kg/s or in $moles/s$) as a function of the concentration difference. it can be seen that the mass transfer coefficient (k) has a similar function as h , the heat transfer coefficient. Equation 56 is also known as Fick's law (on the left). When comparing with Fourier's law (on the right), it seems again as if they are very similar. The diffusion coefficient (D_a) and the thermal conductivity λ 'overlap' in all equations.

$$\phi m'' = -D_a \frac{dc}{dx} \quad \phi q'' = -\lambda \frac{dT}{dx} \quad (56)$$

$$Sh = \frac{kD}{D_a} \quad Nu = \frac{hD}{\lambda} \quad (57)$$

9 Diffusion flow in mass balances

As an example, we take the dissolution of a solid sphere inside a large bin of water. The soluble compound is slowly diffusing from the sphere into the water layer. The concentration at the interface between the solid sphere and water is the saturated concentration of the component in water (or solubility), C_s . To investigate how the diameter of the sphere changes over time due to diffusion of mass, we consider the mass balance:

$$\frac{dV}{dt} = -kA\Delta C \quad (58)$$

ΔC is the driving force for the mass transport which is in this case the difference between C_s and the concentration at a very large distance from the sphere (C_∞), which can assumed to be 0.

A is the area of the sphere (m^2) as that is the area that is subject to mass transport. The area depends on the diameter and the volume depends on the diameter as well, so commonly the terms are rewritten for the diameter:

$$\frac{1}{6}\pi\rho\frac{dD^3}{dt} = -k\pi D^2\Delta C \quad (59)$$

We apply the chain rule to rewrite the derivative of D^3 with respect to t:

$$\frac{dD^3}{dt} = \frac{dD^3}{dD} \frac{dD}{dt} = 3D^2 \frac{dD}{dt} \quad (60)$$

When substituting this into equation 59, we obtain:

$$\frac{1}{2}\pi D^2 \rho \frac{dD^3}{dt} = -k\pi D^2 \Delta C \quad (61)$$

Lastly, k is also a function of the diameter. To show this we introduce the Sherwood number:

$$Sh = \frac{kD}{D_a} \quad k = \frac{Sh D_a}{D} \quad (62)$$

Just like the Nusselt number, the Sherwood can also be determined from Reynolds, by the following formulas.

For spheres:

$$Sh = 2.0 + 0.66Re^{0.5}Sc^{0.33} \quad 10 < Re < 10^4 \quad Sc > 0.7 \quad (63)$$

and in tubes:

$$Sh = 0.027Re^{0.8}Sc^{0.33} \quad 10 < Re < 10^4 \quad Sc > 0.7 \quad (64)$$

Note that if Reynolds is lower than 10, it can be assumed that the Sherwood number is equal to 2 for spheres! From here, the balance can be further developed and finally integration can be applied by using the boundary conditions: $t = 0 \rightarrow d = D_0$, $t = t \rightarrow d = D$. This allows you obtain the diameter as a function of time.

10 Practice Questions

Q1

A PhD student of the department of organic chemistry recently developed a synthesis route for biodegradable surfactants on the basis of sugars. They asked a student in chemical engineering to estimate what the possibilities are for a (semi-) industrial process in a stirred tank reactor of 1.25 m^3 . The student determined the rotational speed and the dissipated power of the stirrer at optimal production circumstances in a geometrical similar (= equally shaped) tank reactor of 10 liters. The motor power P_o (W) is a function of the rotational speed of the stirrer N (s^{-1}), the density of the medium ρ (kg/m^3), the viscosity of the medium η (Pa s) and the diameter of the reactor D (m). The experiments show that optimal conditions can be reached in the 10 liter reactor at a stirrer speed of 150 RPM and at a motor power of 70 Watt.

- (a) How many dimensionless numbers do you expect you need to describe the power of the stirrer and why?
- (b) Give a dimensionless expression for P_o .
- (c) What do you recommend for the stirrer speed in the production reactor?
- (d) What motor power is needed in the production reactor?

Q2

A chemical engineer wants to pump water from tank A to tank B and he wonders what size pump should be installed. The water level in tank B is 3 m higher than that in tank A. Both tanks are open to the atmosphere and they are connected by a 15 m long pipeline with an internal diameter of 10 cm. In this pipe line there are two sharp bends. The friction factor for the pipe is given by: $f = 0.1 \text{ Re}^{-1/4}$. The friction loss factor (K_W) for sharp bends is estimated as 1.3. The process water has a dynamic viscosity of 0.001 Pa s, a density of 1000 kg/m^3 and a specific heat of 4.2 $\text{kJ}/(\text{kg}\cdot\text{K})$.

- (a) Calculate the pumping power needed for a throughput of 20 liter per second.
- (b) Calculate the maximum rise in water temperature due to the pumping.
- (c) How much power can be saved by using a pipe with a perfect smooth wall?

Q3

Due to the wasp plague last summer a pharmaceutical industry wants to introduce a new anti-wasp device (called AWD). This AWD is a ball consisting completely of an active ingredient that slowly evaporates. The vapor pressure of the active ingredient at ambient temperature is 0.05 mm Hg. An AWD ball has initially a diameter of 1 cm and it hangs in a (very) large room with stagnant air (no air movement)

- (a) Show that the concentration of the active ingredient in the air at the surface of the AWD-ball (c^*) equals 0.00273 mol/m^3 .
- (b) Calculate the mass transfer coefficient just after the AWD ball has been placed.
- (c) Calculate the molar flow rate due to evaporation.

- (d) Give an expression for the diameter as a function of time.

Diffusion coefficient of the active ingredient in air: $D_A = 0.7 \times 10^{-5} \text{ m}^2/\text{s}$.

Density of the active ingredient: $\rho_A = 1150 \text{ kg/m}^3$.

Molar mass of the active ingredient: $M_a = 128 \text{ kg/kmol}$.

Q4

When driving a car (1.6 m wide, 1.3 m high, 950 kg) at a steady 105 km/h on a highway (density of air = 1.2 kg/m³) on a windless day, it is noticed that the time required to decelerate to 95 km/h is 5 s.

- (a) Estimate the maximum value of the drag coefficient (drag coefficient depends on velocity).
- (b) Driving steadily, which percentage of the petrol used (10 l/100 km) is converted into mechanical energy, if the heat of combustion is 11000 kcal/kg (density of petrol = 700 kg/m³)? (1kcal = 4.2 kJ)
- (c) What did you neglect?

Q5

What is the terminal velocity of a spherical steel particle, 0.3 mm in diameter, settling in an oil of density 820 kg/m³ and dynamic viscosity of 0.01 Pa.s? The density of steel is 7870 kg/m³.

Q6

A flat oven wall consists of a layer (0.114 m) of a special high temperature resistant material ($\lambda = 0.1384 \text{ W/(m K)}$) and a conventional red brick outer layer (0.229 m, $\lambda = 1.384 \text{ W/(m K)}$). The temperature in the oven is 760°C and the external wall temperature is 76.5 °C.

- (a) Calculate the heat loss of the oven per m^2 surface.
- (b) What is the temperature at the interface between the two layers?

Q7

A non-isolated steel pipe of 100 m is used to transport steam of a temperature of 180°C. The inner tube diameter is 70 mm, the outer diameter is 75 mm. The temperature of the air around the tube is 30°C. The heat transfer coefficient inside and outside of the tube are 3000 W/(m² K) and 1500 W/(m²K), respectively. Thermal conductivity of steel: 45 W/(m.K).

- (a) Calculate the heat loss from the pipe to the surroundings.
- (b) Make a schematic drawing of the temperature profile between the inside of the tube and the outside air.

Q8

A very long horizontal heat exchanger for warming up liquids consists of internally smooth pipes which are kept at constant temperature. The flow through the pipes is turbulent.

If the pressure drop is doubled per unit of length of the pipes because of changes in flow throughput, by which factor does the heat transfer coefficient increase?

Q9

A reaction is carried out in a PFR where A is converted into P. The diameter of the PFR is 30 cm and it has a length of 10 m. $\phi_v = 8$ L/s, $k = 0.1$ m³/mol.s and the inlet concentration ($C_{a,0}$) = 2 mol/m³

- a) Derive the concentration profile in a PFR for a 2nd order kinetics in A.
- b) What is the conversion of A at the outlet of the PFR?
- c) Suppose we have a CISTR with the same volume as the PFR. If the same reaction would be carried out in a CISTR, would the conversion of A be higher or lower than in a PFR?

Solutions of the practice exercises will be sent by E-mail to participants of the support sessions. Alternatively, Request for solutions by sending an E-mail to internalaffairs@tbvlugus.nl