General Instructions

- Please print and sign the FSE honor code page, scan it and put it in your nestor dropbox folder BEFORE starting the exam. If you do not have the capacity to print, use a blank page and recopy the words on the sheet.
- Please put your exam in your nestor dropbox folder when completed. Students can find their Nestor Dropbox folder as follows: Go to the main Nestor page of your course Unfold the folder labeled with the course name Click the item “Dropbox” Click on the folder with your name
- Please use the cover sheet on the next page for your exam. If you do not have the capacity to print, use a blank page and recopy the words on the sheet.
- Please also send your exams to a.m.s.waters@rug.nl.

  Emails should have the title ”Calculus 2 for IEM final examination”

  The body of the email should have your full name (last name, first) and student ID.

FSE honor code instructions

Declaration of the Board of Examiners The Board of Examiners has allowed the conversion of your exam into a take-home exam. This conversion comes with additional provisions.Here are the provisions that are relevant to you sitting the exam:

1. You are required to sign the attached pledge, swearing that your work has been completed autonomously and using only the tools and aids that the examiner has allowed you to use.

2. Attempts at cheating, fraud or plagiarism will be seen as attempts to take advantage of the Corona crisis and will be dealt with very harshly by the board of examiners.

3. The board of examiners grants your examiner the right to conduct a random sampling. If you are selected for this sample, you may be required to conduct a discussion (digitally, using audio and video) in which you are asked to explain and/or rephrase (some of) the answers you submitted for the take-home exam.

FSE honor code

I, ……………......................... (enter your name and student number here) have completed this exam myself and without help from others unless expressly allowed by my lecturer. I have come up with these answers myself. I understand that my fellow students and my lecturers are all doing their best to do their work as well as possible under the unusual circumstances of the Corona pandemic, and that any attempt by myself or my fellow students to use these circumstances to get away with cheating would be undermining those efforts and the necessary trust that this moment calls for.

Signature:

Cover sheet for the exam

Name:

Student ID:

Signature:

Calculus 2 IEM Final Exam 2020

Show all work for full credit. Maximum score is 100 points, all questions are worth equal point values. Students receive 10 points are for following all of the instructions. Exam time is 3+1=4 hours (including scanning time). Please state clearly any theorems from class you use. No calculators you can use the given formula sheet, your book, and notes. Good luck!

(1) Find the limits or show that they do not exist:
$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^2-y}{x^2+y^2+z^2}$$
$$\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+y^2}$$

(2) a) Write the linearisation $L(x,y)$ of the function
$$f(x,y)=x^3y^{\frac{4}{5}}+\sqrt{4x^2+21y^2}$$
at the point $(1,1)$
b) Use this to approximate $f(1.1,0.8)$.

(3) a) Calculate the directional derivative of
$$f(x,y,z)=x^2+e^yz+yxz+17$$
at the point $(5,1,3)$ in the direction of the origin.
b) What is the minimum value of the the directional derivative and why?

(4) a) Let $f(x,y)$ be a function of two variables. Let
$$f(x,y)=xy\ln\left(x^2+y^2-1\right)$$
calculate f_{yx} and f_{xy}.
b) State clearly how Clairaut's theorem applies to the mixed partials.

(5) a) Let the vector field F be given by
$$F=\frac{x\vec{i}+y\vec{j}}{x^2+y^2+1}$$
show that there exists a vector field f such that $\nabla f=F$.
b) Let $\mathcal{C}$ be the ellipse $4x^2+9y^2=36$ orientied clockwise. Compute $\int\limits_{\mathcal{C}} F\cdot dr$.

(6) Find the local minimum and maximum of the function
$$f(x,y)=x^3+y^3-12xy \tag{0.1}$$

(7) Consider the surface
$$x^2+\frac{y^2}{4}=z^2$$
and the helical curve segment
$$r(t)=<\cos(t),2\sin(t),t> \quad 0\le t\le 2$$

a) Find the unique point of intersection.
b) Find the cosine of the angle between the tangent plane to the surface and tangent vector the helical curve at the point.

(8) Suppose that $x = g(t)$ and $y = h(t)$ and that

$$x \ln y = 2$$

Find an equation which relates

$$\frac{dx}{dt} \quad \text{and} \quad \frac{dy}{dt}$$

in terms of x and y.

(9) a) Find the extrema and classify them of

$$f(x, y) = e^{-(x^2+y^2)}$$

subject to the constraint $x + y = 1$.
b) Why does the number of extrema that you found in part a) not contradict the extreme value theorem?